Sample: Geometry - Miscellanea

Question 1

John likes to play baseball with his friends in the local playing fields. Last week John hit his best shot ever. The trajectory of the baseball after he hit it can be modeled by the quadratic equation

$$y = -\frac{x^2}{40} + \frac{31x}{40} + \frac{4}{5},$$

where y represents the height in metres of the baseball above the ground, and x represents the horizontal distance in metres of the baseball from the position where it was struck by John. Assume that the surface of the playing field is horizontal.

(a) The graph of $y = -\frac{x^2}{40} + \frac{31x}{40} + \frac{4}{5}$ is a parabola.

(i) Is the parabola u −shaped or n −shaped? How can you tell this from the equation?

(ii) Use algebra to find the x −intercepts and y −intercept.

(iii) Find the equation of the axis of symmetry, explaining your method. Use this information to find the coordinates of the vertex, giving your answers to two decimal places.

(iv) Provide a sketch of the graph of the parabola, either by hand or by using Graphplotter.

Solution:

(i) The graph of a quadratic equation $y = ax^2 + bx + c$ is a parabola.

If a>0, then the parabola has a minimum point and it opens upwards (u-shaped)

If a<0, then the parabola has a maximum point and it opens downwards (n-shaped)

$$y = -\frac{x^2}{40} + \frac{31x}{40} + \frac{4}{5}$$

$a = -\frac{1}{40} < 0$, so **the parabola is $\boldsymbol{n}$ −shaped**

(ii) When $x = 0$, then $y = \frac{4}{5}$, so **$\boldsymbol{y}$ −intercept is $\left(\mathbf{0}, \frac{\mathbf{4}}{\mathbf{5}}\right)$.**

When $y = 0$, then

$$-\frac{x^2}{40} + \frac{31x}{40} + \frac{4}{5} = 0$$

$$x^2 - 31x - 32 = 0$$

$$x = \frac{31 \pm \sqrt{961 + 128}}{2} = \frac{31 \pm 33}{2}$$

$x = -1, x = 32$, so **$\boldsymbol{x}$ −intercept are $(-\mathbf{1}, \mathbf{0})$ and $(\mathbf{32}, \mathbf{0})$.**

(iii) For a quadratic function in standard form, $y = ax^2 + bx + c$, the axis of symmetry is a vertical line $x = -\frac{b}{2a}$

$$a = -\frac{1}{40}, b = \frac{31}{40}$$

$$x = -\frac{\frac{31}{40}}{-\frac{2}{40}} = \frac{31}{2} = 15.5$$

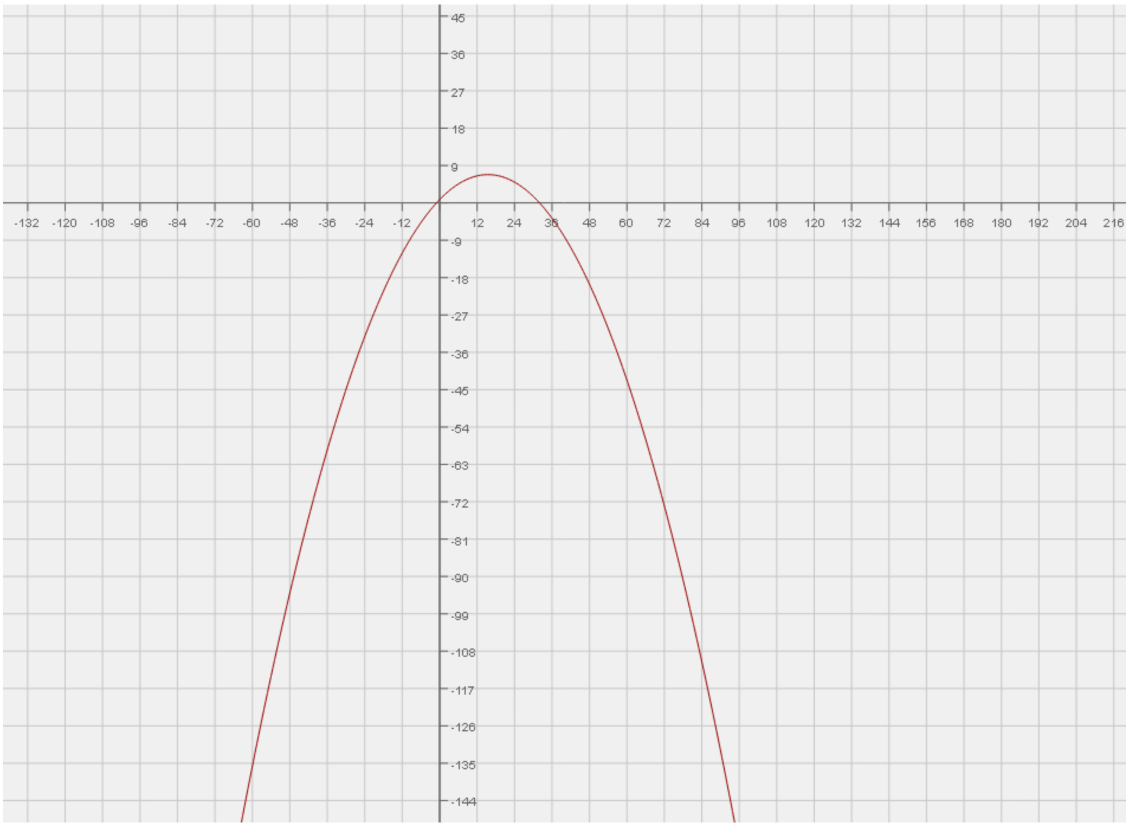
$$x = 15.5$$

Vertex is on the axis of symmetry, so y coordinate of the vertex is:

$$y = -\frac{15.5^2}{40} + \frac{31 \cdot 15.5}{40} + \frac{4}{5} = 6.81$$

Thus, the **vertex is** $(\mathbf{15.5, 6.81})$.

(iv)



(b) In this part of the question, you are asked to consider the trajectory of the baseball modeled by the equation $y = -\frac{x^2}{40} + \frac{31x}{40} + \frac{4}{5}$ in conjunction with the results that you found in part (a).

(i) Find the height of the baseball when it was 5 metres horizontally from the position where John hit it.

(ii) Use your answer to part (a)(iii) to find the maximum height reached by the baseball.

(iii) What does the y –intercept represent in the context of this model?

(iv) Assuming that the baseball was not caught, how far was it horizontally from where John hit it when it first landed on the ground? Explain your answer.

Solution:

(i) x represents the horizontal distance in metres of the baseball from the position where it was struck by John , so $x = 5$.

When $x = 5$, then

$$y = -\frac{5^2}{40} + \frac{31 \cdot 5}{40} + \frac{4}{5} = 4.05$$

The height of the baseball was **4.05 metres.**

(ii) The maximum height reached by the baseball is equal y coordinate of the vertex

6.81 metres.

(iii) y –intercept represents the height at which the baseball struck by John

(iv) When John hit baseball $x = 0$, when baseball first landed on the ground $y = 0$(the second x-intercept $(32{,}0)$), so baseball was far horizontally **32 meters.**

Question 2

(a) Use the quadratic formula to solve the equation

$5m^2 - 11m + 3 = 0.$

Give your answers correct to two decimal places.

Solution:

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$5m^2 - 11m + 3 = 0$

$$x = \frac{11 \pm \sqrt{11^2 - 4 \cdot 5 \cdot 3}}{2 \cdot 5} = \frac{11 \pm \sqrt{121 - 60}}{10} = \frac{11 \pm \sqrt{61}}{10}$$

$\boldsymbol{x = 1.88}$ **and** $\boldsymbol{x = 0.32}$

(b) This part of the question concerns the quadratic equation

$3x^2 + 6x + 13 = 0.$

(i) Find the discriminant of the quadratic expression $3x^2 + 6x + 13$.

(ii) What does this tell you about the number of solutions of the equation? Explain your answer briefly.

(iii) What does this tell you about the graph of $y = 3x^2 + 6x + 13$?

Solution:

(i) If the quadratic expression is $ax^2 + bx + c$, then the discriminant is:

$\Delta = b^2 - 4ac$

$3x^2 + 6x + 13$

$\Delta = 6^2 - 4 \cdot 3 \cdot 13 = -120$

(ii) $\Delta = -120 < 0$, so $\sqrt{\Delta}$ is not a real number, so there are not solutions in real numbers.

(iii) The graph of $y = 3x^2 + 6x + 13$ has not x –intercepts.

(c)

(i) Write the quadratic expression $x^2 - 42x - 7$ in completed-square form.

(ii) Use the completed-square form from part (c)(i) to solve the equation $x^2 - 42x - 7 = 0$, leaving your answer in exact (surd) form.

(iii)Use the completed-square form from part (c)(i) to write down the vertex of the parabola $y = x^2 - 42x - 7$.

Solution:

(i) $x^2 - 42x - 7 = x^2 - 42x + \left(\frac{42}{2}\right)^2 - \left(\frac{42}{2}\right)^2 - 7 = \boldsymbol{(x - 21)^2 - 448}$

(ii) $x^2 - 42x - 7 = 0$

$(x - 21)^2 - 448 = 0$

$(x - 21)^2 = 448$

$(x - 21) = \pm\sqrt{448} = \pm 8\sqrt{7}$

$\boldsymbol{x = 21 \pm 8\sqrt{7}}$

(iii) $y = x^2 - 42x - 7$

$y = (x - 21)^2 - 448$

$y + 448 = (x - 21)^2$

If the equation of parabola is $4p(y - k) = (x - h)^2$, then the coordinates of the vertex is (h, k)

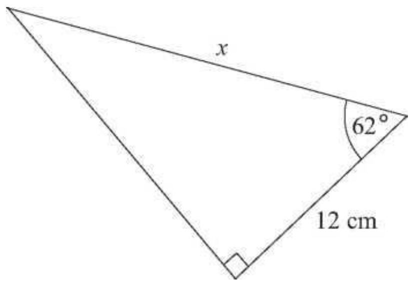
$k = -448, h = 21$

So the coordinates of the vertex is $(\mathbf{21}, \mathbf{-448})$

Question 3

(a) Find the length of the side marked x in the triangle in Figure 1,

giving your answer correct to the nearest cm.



Solution:

$$\frac{12}{x} = \cos 62°$$

$$\boldsymbol{x} = \frac{12}{\cos 62°} = \boldsymbol{26\ cm}$$