Sample: Microeconomics - Hicksian Demand Function

Problem #1

a) Solution

Utility level $u(x_1, x_2) = u$. Then the Hicksian demand function for each good solves the expenditure minimization problem:

$$\min_{x_1, x_2} e(p_1, p_2, u) = p_1 x_1 + p_2 x_2$$

Subject to:

$$(x_1)^\beta (x_2)^{1-\beta} = u \qquad (1)$$

We have the system of equations:

$$\begin{cases} \dfrac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \dfrac{p_1}{p_2} \\ (x_1)^\beta (x_2)^{1-\beta} = u \end{cases} \Rightarrow$$

$$\Rightarrow \frac{\beta \cdot (x_2)^{1-\beta} (x_2)^\beta}{(1-\beta) \cdot (x_1)^{1-\beta} (x_1)^\beta} = \frac{\beta \cdot x_2}{(1-\beta) \cdot x_1} = \frac{p_1}{p_2} \Rightarrow x_2 = \frac{p_1 \cdot (1-\beta) \cdot x_1}{p_2 \cdot \beta} \qquad (2)$$

In this case we will have:

$$(x_1)^\beta (x_2)^{1-\beta} = u \Rightarrow (x_1)^\beta \left(\frac{p_1 \cdot (1-\beta) \cdot x_1}{p_2 \cdot \beta} \right)^{1-\beta} = u \Rightarrow$$

$$\Rightarrow x_1 = \frac{u \cdot (p_2 \cdot \beta)^{1-\beta}}{\left(p_1 \cdot (1-\beta)\right)^{1-\beta}}$$

Then we can find x_2 from (2):

$$x_2 = \frac{p_1 \cdot (1-\beta) \cdot x_1}{p_2 \cdot \beta} = \frac{p_1 \cdot (1-\beta)}{p_2 \cdot \beta} \cdot \frac{u \cdot (p_2 \cdot \beta)^{1-\beta}}{\left(p_1 \cdot (1-\beta)\right)^{1-\beta}} = \frac{u \cdot \left(p_1 \cdot (1-\beta)\right)^{\beta}}{(p_2 \cdot \beta)^{\beta}}$$

The Hicksian demand function for each good:

$$x_1^h = \frac{u \cdot (p_2 \cdot \beta)^{1-\beta}}{\left(p_1 \cdot (1-\beta)\right)^{1-\beta}}$$

$$x_2^h = \frac{u \cdot \left(p_1 \cdot (1-\beta)\right)^{\beta}}{(p_2 \cdot \beta)^{\beta}}$$

Answer: The Hicksian demand function for each good: $x_1^h = \frac{u \cdot (p_2 \cdot \beta)^{1-\beta}}{\left(p_1 \cdot (1-\beta)\right)^{1-\beta}}$ and $x_2^h = \frac{u \cdot \left(p_1 \cdot (1-\beta)\right)^{\beta}}{(p_2 \cdot \beta)^{\beta}}$.

b) The expenditure function:

$$e(p_1, p_2, u) = p_1 x_1^h + p_2 x_2^h = \frac{p_1 \cdot u \cdot (p_2 \cdot \beta)^{1-\beta}}{\left(p_1 \cdot (1-\beta)\right)^{1-\beta}} + \frac{p_2 \cdot u \cdot \left(p_1 \cdot (1-\beta)\right)^{\beta}}{(p_2 \cdot \beta)^{\beta}} =$$

$$= \frac{p_1^{\beta} \cdot u \cdot (p_2 \cdot \beta)^{1-\beta}}{(1-\beta)^{1-\beta}} + \frac{p_2^{1-\beta} \cdot u \cdot \left(p_1 \cdot (1-\beta)\right)^{\beta}}{(\beta)^{\beta}} =$$

$$= \frac{p_1^{\beta} \cdot u \cdot (p_2)^{1-\beta} \cdot \beta + p_2^{1-\beta} \cdot u \cdot (p_1)^{\beta} \cdot (1-\beta)}{(\beta)^{\beta}(1-\beta)^{1-\beta}} = \frac{u \cdot p_1^{\beta} \cdot p_2^{1-\beta}}{(\beta)^{\beta}(1-\beta)^{1-\beta}}$$

We received that the expenditure function is:

$$e(p_1, p_2, u) = \frac{u \cdot p_1^{\beta} \cdot p_2^{1-\beta}}{(\beta)^{\beta}(1-\beta)^{1-\beta}}$$

Answer: $e(p_1, p_2, u) = \frac{u \cdot p_1^{\beta} \cdot p_2^{1-\beta}}{(\beta)^{\beta}(1-\beta)^{1-\beta}}$.

Problem #2

a) We have that $m = p_1x_1 + p_2x_2$ and $MRS = \frac{p_1}{p_2}$. Also we know that the price of good 2 $p_2 = \$1$. Utility function is $u(x_1, x_2) = \sqrt{x_1} + x_2$.

We have that:

$$\frac{U_{x_1}}{U_{x_2}} = \frac{p_1}{p_2} = |p_2 = \$1| = p_1 \Rightarrow \frac{1}{2\sqrt{x_1}} = p_1$$

We can find x_1:

$$\frac{1}{4p_1^2} = x_1$$

$$m = p_1x_1 + p_2x_2 = \frac{1}{4p_1} + x_2 \Rightarrow x_2 = m - \frac{1}{4p_1}$$

The ordinary demand functions for two goods:

$$x_1 = \frac{1}{4p_1^2}$$

$$x_2 = m - \frac{1}{4p_1}$$

<u>Answer:</u> The ordinary demand functions for two goods: $x_1 = \frac{1}{4p_1^2}$ and $x_2 = m - \frac{1}{4p_1}$ and utility: $u = \frac{1}{2p_1} + m$.

b) The utility function is quasi-linear and the consumer surplus in our case will be equal:

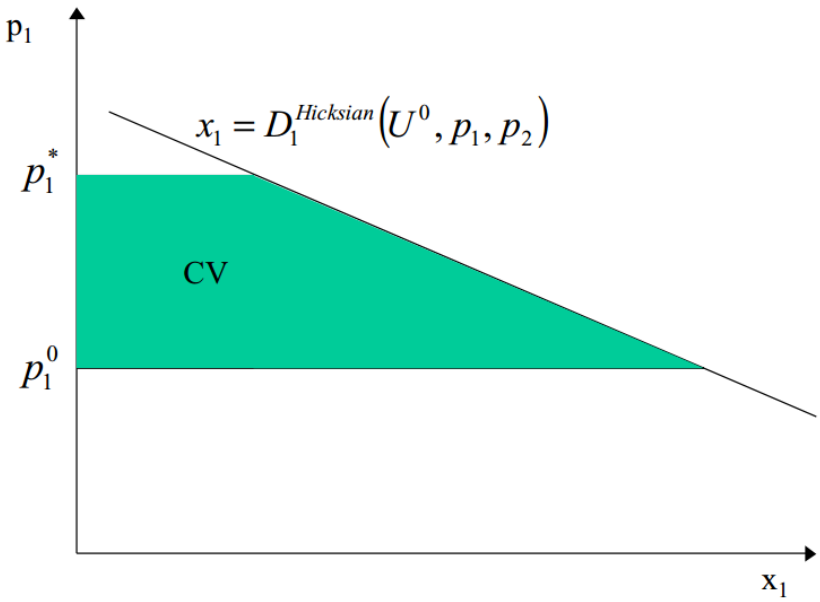
$$CS = p_1 \cdot x_1 = p_1 \cdot \frac{1}{4p_1^2} = \frac{1}{4p_1}$$

As we can see, when the price for good 1 will increase, the utility function will decrease.

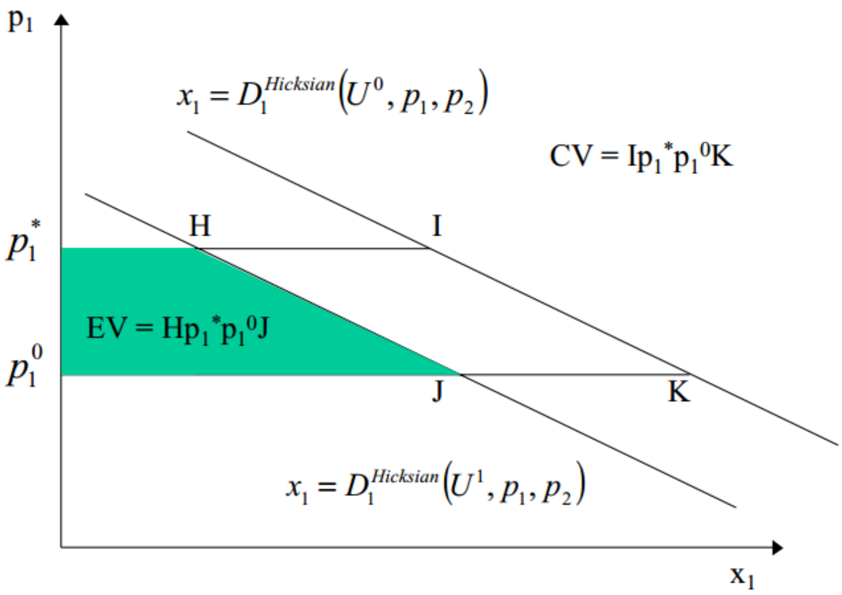
Answer: $CS = \frac{1}{4p_1}$ and the consumer surplus decreases when the price of good 1 increases.

Problem #3

Compensating variation is the area to the left of the Hicksian demand curve. If we know that only price of good 1 changes, then we have:



Equivalent variation is the maximum amount the consumer would be willing to pay to avoid a price change. At old prices EV is the amount of the income necessary to get tge new level of utility. EV is also the area to the left of the Hicksian demand curve, but compared to CV, it's a different Hicksian demand curve. The one associated with new level of utility.



As you can see from the picture above, compensating variation associated with the Hicksian demand curve when utility level is U^0 and the equivalent variation associated with the Hicksian demand curve with utility level U^1.

So, the equivalent and compensative variation will be different in case if the price of good 1 changes by the same amount.

Problem #4

a) We have two periods and next data:

Income:

- for the period 1: m_1;
- for the period 2: m_2.

Consumption:

- for the period 1: c_1;
- for the period 2: c_2.

Interest rate: r.

We will take that her savings for the period 1: S_1.

Utility function: $u(c_1, c_2) = min\{c_1, c_2\}$.

So, we have:

$$c_1 + S_1 \leq m_1 \qquad (3)$$

$$c_2 \leq m_2 + S_1 \cdot (1 + r) \qquad (4)$$

From (3) and (4) we will find constraint:

$$S_1 = m_1 - c_1 \qquad (5)$$

$$c_2 = m_2 + (m_1 - c_1) \cdot (1 + r) \qquad (6)$$

From (5) and (6) we will have:

$$c_2 - m_2 = (m_1 - c_1) \cdot (1 + r) \Rightarrow m_1 - c_1 = \frac{c_2 - m_2}{(1 + r)} \Rightarrow m_1 - c_1 = \frac{c_2}{(1 + r)} - \frac{m_2}{(1 + r)}$$

$$\Rightarrow c_1 + \frac{c_2}{(1 + r)} = m_1 + \frac{m_2}{(1 + r)}.$$

The left hand side shows the present value expenditure and right hand side depicts the present value income respectively.

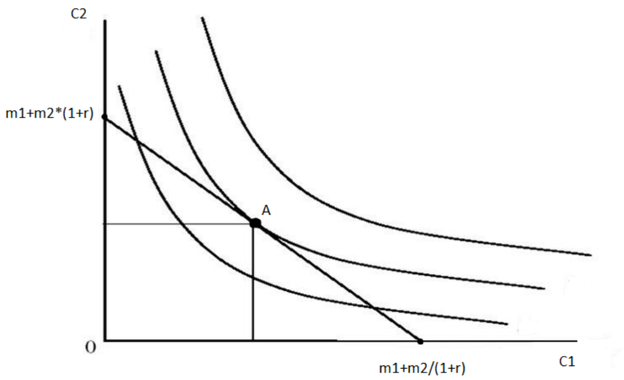
Now we will have the problem:

$u(c_1, c_2) = min\{c_1, c_2\}$

Subject to:

$$c_1 + \frac{c_2}{(1+r)} = m_1 + \frac{m_2}{(1+r)}.$$

b) We have point A as a solution:



c) If the consumer's income for both periods double, then, I thing, that she will spend more money than before this and maybe her savings will be greater, but not doubled. She knows that she has stable increase in income (as she always has her skills), so she know that in future he will have more money and she wouldn't save more and more for the future. Her consumption will increase because of consumption smoothing motives.

d) Of course her consumption will increase because of the smoothing motives, but compared with point c) this increase will be smaller. And it will lead to increasing in savings.