Sample: Macroeconomics - Growth Models

Question #1

The Solow growth model breaks the growth of economies down into basics: it starts with production function $Y = F(K, L)$ and then breaks it into per worker view:

$$\frac{Y}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) \Rightarrow y = f(k)$$

In this case we receive that k is the amount of capital per worker and the amount of the output per worker – y. As we know the slope of the function is the rate of change (or velocity) and in this case the slope of the function is the change in output per worker due to the one unit increase in capital per worker and it's equal to the marginal product of capital - MPK.

The slope of the output per worker line is $f_0(k) = MPK$. Due to the decreasing marginal productivity of capita, this is decreasing in y, making $f(k)$ a concave function.

Individuals consume whatever they don't save, where s (which is somewhere between 0 and 1) is a saving rate. We have that consumption in this case will be equal to:

$$c = (1 - s) \cdot y.$$

So, in total we have that all output of each worker is either allocated to consumption or to investment:

$$y = c + i.$$

And we see that the investment is the saved output: $i = s \cdot y$.

The population growth (n) doesn't affect the amount of capital (K), but it will decrease amount of capital per worker. Depreciation (δ) is the rate at which capital wears out. These two factors are eating away at our capital per worker. If we want to retain an unchanging level of capital per working k over time, we have to invest enough to create new capital to offset this loss over time. Thus, to maintain a state where the per-wo rker capital is constant over time, we should have that:

$$\Delta k = s \cdot f(k) - k \cdot (\delta + n) = 0 \Rightarrow s \cdot f(k^*) = k^* \cdot (\delta + n)$$

The symbol $*$ indicates the "steady state" values, or the values where per-worker capital is constant over time.

We can see that as our per-worker capital gets larger we need more investments to maintain $\Delta k = 0$.

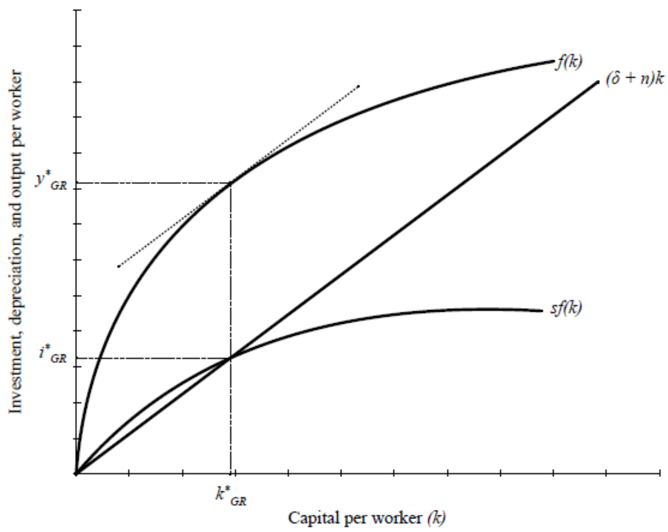
The economy will always work itself to a steady state point. If $s \cdot f(k) > k \cdot (\delta + n)$, then capital stock will grow. If $s \cdot f(k) < k \cdot (\delta + n)$, then capital stock will shrink. Only when the two are equal will there be no further adjustment to capital stock in the economy.

There is an infinite number of possible “steady states”, some higher than others. Which steady state our economy is in depends on where the investments curve $s \cdot f(k)$ meets “loss” curve $k \cdot (\delta + n)$, which in turn depends on the savings rates in the economy. To find the optimal point we need to use the Golden Rule.

The more people consume, the happier they are. If we want people to be as happy as possible, our aim is to maximize consumption per worker c. The steady state associated with that particular outcome is called the “Golden Rule” (GR) steady state. The steady state:

$$c^* = s \cdot f(k^*) - k^* \cdot (\delta + n).$$

While higher levels of capital mean higher levels of output, they also mean more capital is being “removed” from the economy each year. If capital stock is below the GR level, the slope of the production function is greater than that of the capital stock curve, and increase in capital per worker has a greater impact on $f(k)$ than on $k \cdot (\delta + n)$ giving us increase in consumption (or in opposite side if we are above th GR level). The GR steady state occurs when $f'(k^*) = MPK = k^* \cdot (\delta + n)$. If MPK is greater than $k \cdot (\delta + n)$, we know that adding capital will increase consumption. If MPK is less than $k \cdot (\delta + n)$, we know that decreasing capital will increase consumption. Maximization of consumption occurs when $f'(k^*) = MPK = k^* \cdot (\delta + n)$. A planner trying to maximize long-run consumption would then aim to get a savings rate corresponding to that particular steady state level of capital.



In the end we can say that the main concept of the golden rule is as capital accumulates, output increases, and thus so does consumption; we sacrifice consumption now for higher consumption for the people of the future.

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Question #2

The country has the following production function:

$$Y = Y(K, L) = K^a \cdot L^b,$$

Where: $Y = output$, $K = capital\ stock$, and $L = labor\ force$,

$a + b = 1,$

$a = 0.356.$

Country doesn't experience any population growth or technological change and 8% of capital depreciates every year. For the future: the country saves 18% of its output per year.

(i) We have that the capital per worker function will be:

$$y = f(k) = \frac{Y}{L} = \frac{K^a \cdot L^b}{L} = \frac{K^a}{L^{1-b}} = \frac{K^a}{L^a} = k^a = k^{0.356}.$$

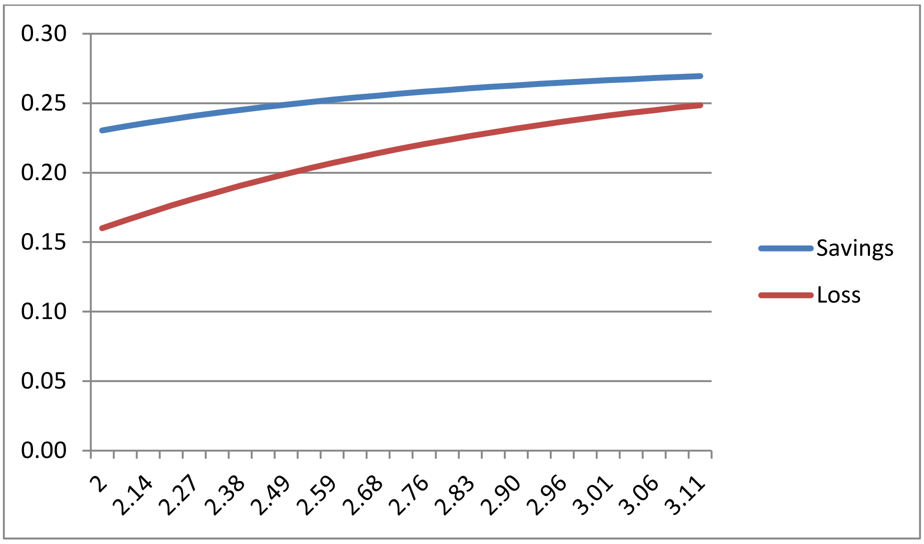
We have:

$$0.18 \cdot (k^*)^{0.356} = 0.08 \cdot k^* \Rightarrow (k^*)^{0.644} = \frac{0.18}{0.08} = 2.25.$$

The table:

Capital	Output	Savings	Loss
2	1,28	0,23	0,16
2,07	1,30	0,23	0,17
2,14	1,31	0,24	0,17
2,20	1,32	0,24	0,18
2,27	1,34	0,24	0,18
2,32	1,35	0,24	0,19
2,38	1,36	0,25	0,19
2,44	1,37	0,25	0,19
2,49	1,38	0,25	0,20
2,54	1,39	0,25	0,20
2,59	1,40	0,25	0,21
2,63	1,41	0,25	0,21
2,68	1,42	0,26	0,21
2,72	1,43	0,26	0,22
2,76	1,43	0,26	0,22
2,79	1,44	0,26	0,22
2,83	1,45	0,26	0,23
2,86	1,45	0,26	0,23
2,90	1,46	0,26	0,23
2,93	1,47	0,26	0,23
2,96	1,47	0,26	0,24
2,99	1,48	0,27	0,24
3,01	1,48	0,27	0,24
3,04	1,49	0,27	0,24
3,06	1,49	0,27	0,25
3,09	1,49	0,27	0,25
3,11	1,50	0,27	0,25

The chart:



We received: $k^* = 3.5$.

(ii) We have:

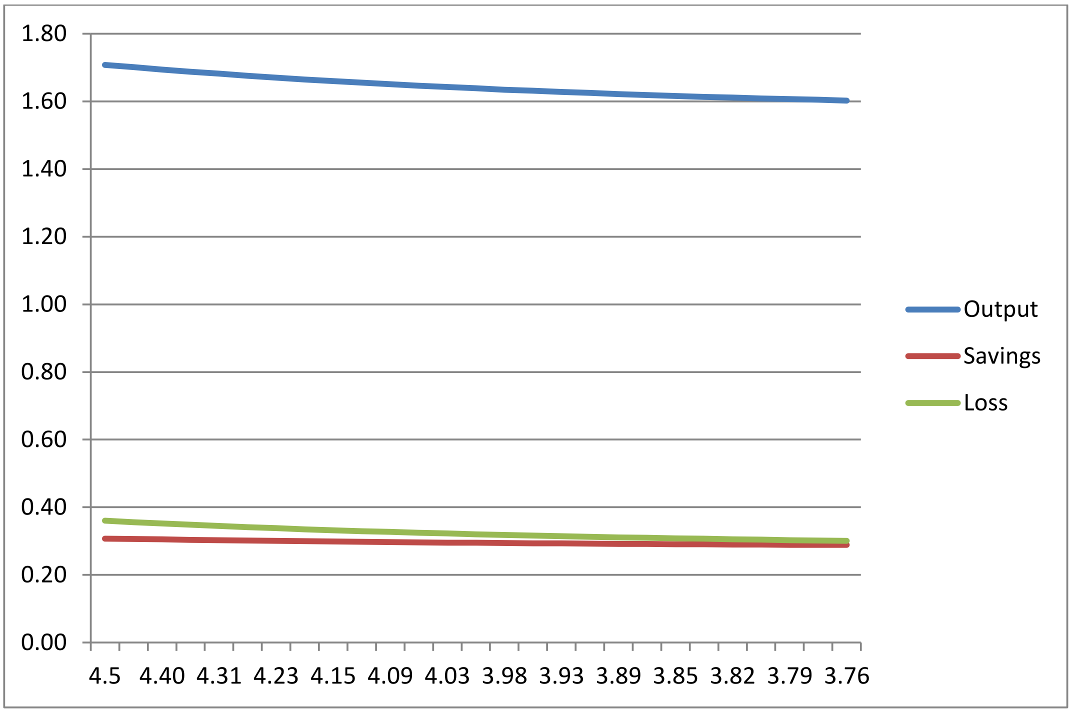
$$k^*_{GOLD} = \frac{0.356}{0.08} = 4.45.$$

The table:

Capital	Output	Savings	Loss
4,5	1,71	0,31	0,36
4,45	1,70	0,31	0,36
4,40	1,69	0,30	0,35
4,35	1,69	0,30	0,35
4,31	1,68	0,30	0,34
4,26	1,68	0,30	0,34
4,23	1,67	0,30	0,34
4,19	1,67	0,30	0,34
4,15	1,66	0,30	0,33
4,12	1,66	0,30	0,33
4,09	1,65	0,30	0,33
4,06	1,65	0,30	0,32
4,03	1,64	0,30	0,32
4,00	1,64	0,29	0,32
3,98	1,63	0,29	0,32
3,95	1,63	0,29	0,32
3,93	1,63	0,29	0,31
3,91	1,62	0,29	0,31
3,89	1,62	0,29	0,31
3,87	1,62	0,29	0,31
3,85	1,62	0,29	0,31
3,83	1,61	0,29	0,31
3,82	1,61	0,29	0,31
3,80	1,61	0,29	0,30
3,79	1,61	0,29	0,30
3,77	1,60	0,29	0,30
3,76	1,60	0,29	0,30

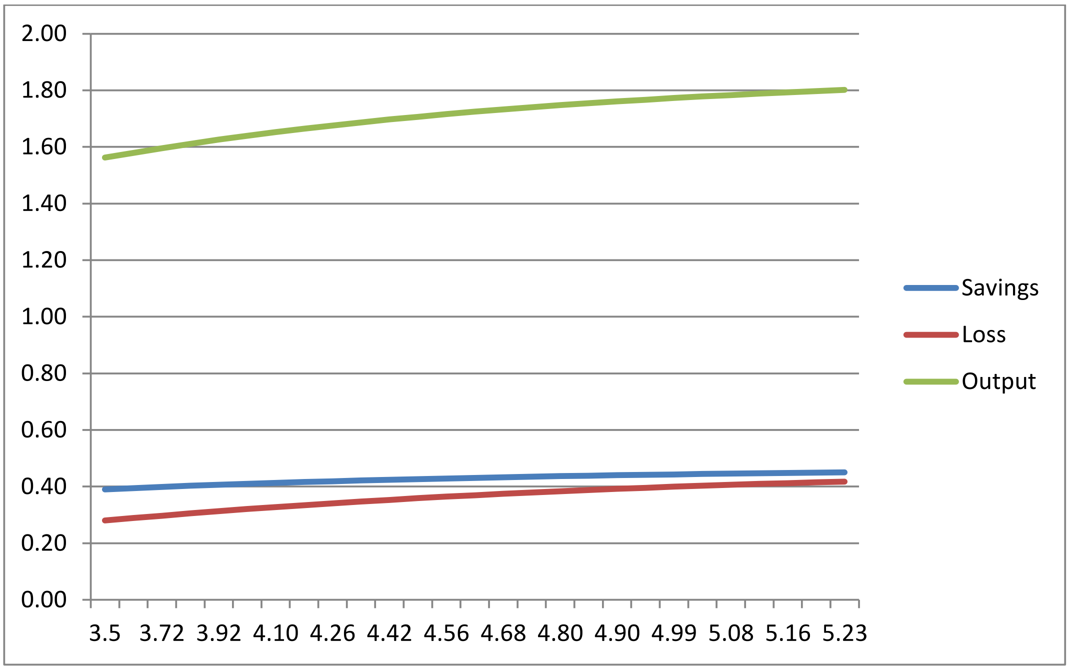
As we see the optimal solution or Golden Rule state is $k = 3.5$.

The chart:


1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
4.5 4.40 4.31 4.23 4.15 4.09 4.03 3.98 3.93 3.89 3.85 3.82 3.79 3.76
Output
Savings
Loss

(iii) We will have:

Capital	Output	Savings	Loss
3,5	1,56	0,39	0,28
3,61	1,58	0,39	0,29
3,72	1,60	0,40	0,30
3,82	1,61	0,40	0,31
3,92	1,63	0,41	0,31
4,01	1,64	0,41	0,32
4,10	1,65	0,41	0,33
4,18	1,66	0,42	0,33
4,26	1,68	0,42	0,34
4,34	1,69	0,42	0,35
4,42	1,70	0,42	0,35
4,49	1,71	0,43	0,36
4,56	1,72	0,43	0,36
4,62	1,72	0,43	0,37
4,68	1,73	0,43	0,37
4,74	1,74	0,44	0,38
4,80	1,75	0,44	0,38
4,85	1,75	0,44	0,39
4,90	1,76	0,44	0,39
4,95	1,77	0,44	0,40
4,99	1,77	0,44	0,40
5,04	1,78	0,44	0,40
5,08	1,78	0,45	0,41
5,12	1,79	0,45	0,41
5,16	1,79	0,45	0,41
5,19	1,80	0,45	0,42
5,23	1,80	0,45	0,42



The output will always rise (savings is greater than depreciation (loss) and the consumption is also increasing).