Sample: Algebra - Factoring quadratics

52. $\mathbf{18z + 45 + z^2}$

Firstly rewrite the equation $18z + 45 + z^2$ putting the largest value first and using the same sign as the original middle value (compare our equation in a standard form $ax^2 + bx + c = 0$):

$$z^2 + 18z + 45 = 0$$

Note that there is not a **GCF** for all the terms. A quadratic trinomial is a trinomial of three terms. We can apply **factoring** method by **grouping**.

Identify the values for $a, b \text{ and } c$

$$a = 1\ b = 18\ c = 45$$

Multiply the leading coefficient$a$, 1, and the constant term, c:

$$1 \cdot (+45) = 45$$

Consider all of the possible factors of this new product:

Factors of +45
$(1) \cdot (45)$
$(3) \cdot (15)$
$(5) \cdot (9)$
$(9) \cdot (5)$

We can note that **prime factors** of product 45 are $3 \times 3 \times 5$. From the list of factors, we find the one pair that adds to the middle term's coefficient, b. For this example, we need to find a sum of 18.

$$3 + 15 = 18$$

Now we can rewrite the middle term, forming two terms, using these two values:

$$z^2 + 3z + 15z + 45$$

Group the first two terms together and group the last two terms together:

$$(z^2 + 3z) + (15z + 45)$$

Notice the plus sign between the two groups.

Factor the **greatest common factor** out of each group:

$$z(z + 3) + 15(z + 3)$$

Notice that the expressions in the parentheses are identical. By **factoring** out the parentheses binomial, we have the answer:

$$(z + 3)(z + 15)$$

We can check getting answer:

$$(z+3)(z+15)$$

We have to multiply expressions in the parentheses:

$$(z+3)(z+15) = z^2 + 3z + 15z + 45 = z^2 + 18z + 45$$

Also we can apply method **factoring** trinomial by **factor** theorem or by using quadratic formula:

$$az^2 + bz + c = 0 \text{ (When } a \neq 0)$$

And they are

$$z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve quadratic equation $z^2 + 18z + 45$ using the quadratic formula.

Here we have $a = 1$ $b = 18$ $c = 45$

$$z_{1,2} = \frac{-18 \pm \sqrt{(18)^2 - 4 \cdot 45}}{2}$$

$$z_{1,2} = \frac{-18 \pm \sqrt{324 - 180}}{2}$$

$$z_{1,2} = \frac{-18 \pm \sqrt{144}}{2}$$

144 is a **perfect square** because $12^2 = 144$

$$z_{1,2} = \frac{-18 \pm 12}{2}$$

To calculate first solution we use "+" sign:

$$z_1 = \frac{-18 + 12}{2} = \frac{-6}{2} = -3$$

To calculate second solution we use "-" sign:

$$z_2 = \frac{-18 - 12}{2} = \frac{-30}{2} = -15$$

The solutions are:

$$z_1 = -3$$

$$z_2 = -15$$

Use formula for factoring quadratic equation:

$$ax^2 + bx + c = a(x_1 - x)(x_2 - x)$$

In our case we have:

$$az^2 + bz - c = a(z_1 - z)(z_2 - z)$$

$$(z + 3)(z + 15) = z^2 + 15z + 3z + 45 = z^2 + 18z + 45$$

Answer: $\mathbf{z^2 + 18z + 45 = (z + 3)(z + 15)}$

78) $a^4b + a^2b^3$

To solve this polynomial is the most appropriate method we can use the Distributive Property to express a polynomial in factored form. Use the Distributive Property to express the polynomial as the product of the **GCF** and the remaining **factor** of each term.

$$a^4b + a^2b^3 = a^2b(a^2 + b^2)$$

The largest monomial that we can factor out of each term isa^2b

Answer: $\mathbf{a^2b(a^2 + b^2)}$

It should be noted that quadratic trinomial can be factored using the following three methods:

- **Factoring** by **grouping**
- completing the square
- Using quadratic formula.

The "x Method" of **factoring** helps us to see the possible combination of numbers that will equal to a given number when multiplied and added together. Trinomial can be factorized by **factor** theorem or by using quadratic formula. Our task was the most appropriate method of **grouping** the polynomial method as a quadratic equation longer the steps of mathematical calculations.

In solving the polynomial fitting method for the calculation is determined by the form of the polynomial, the presence of the **greatest common factor**, solutions by dividing the possibility of a **perfect square**.

With any method of calculation should be noted the importance of validation solutions.

66. $\mathbf{8x^2 - 2xy - y^2}$

To solve this problem, we use the "AC" method of **factoring**. The "AC" method or **factoring** by **grouping** is a technique used to **factor** trinomials. A trinomial is a mathematical expression that consists of three terms. The general form of trinomial in two variables is $x^2 + bxy + cy^2$. This trinomial has two variables, x and y. From factorization theorem, factors of these trinomials are:

$$x^2 + bxy + cy^2 = x^2 + (m + n)xy + (mn)y^2 = (x + my)(x + ny)$$

Let us **factor** the trinomial $8x^2 - 2xy - y^2$

Identify the values for $a, b\ and\ c$

$$a = 8\ b = -2\ c = -1$$

Firstly, we need to find product of $a \cdot c = \left(8 \cdot (-1)\right) = -8$

-8	**Sum**
$(-1)\ (8)$	7
$(-2)\ (4)$	2
$(-4)\ (2)$	(-2)
$(-8)\ (1)$	(-7)

As a result of the calculations, we can determine that the solutions of a polynomial in our case, we fit a pair of numbers -4 and 2, they give us sum of value b. Getting the values put into the general expression of the polynomial, as a result we obtain the following.

We can write $8x^2 - 4xy + 2xy - y^2 = 4x(2x - y) + y(2x - y)$

Isolate similar terms and **factor** out the **greatest common factor (GCF)**. **Factor** out $(2x - y)$ and rewrite:

$$(4x + y)(2x - y)$$

After all conducted mathematical operations we can to verify the obtained expression:

$(4x + y)(2x - y) = 8x^2 - 4xy + 2xy - y^2 = 8x^2 - 2xy - y^2$

The check result received initial expression of a polynomial.

Answer: $\mathbf{8x^2 - 2xy - y^2 = (4x + y)(2x - y)}$