Sample: Algebra - Equations

Question 1

(a) Transpose the following formula to make v the subject

$$f = \frac{uv}{u+v}$$

(b) Solve the following equation to find the value of x:

$$(3.4)^{2x+3} = 8.5$$

(c) In formula $\theta = Ve^{-\frac{Rt}{L}}$, the value of $\theta = 58, V = 255, R = 0.1$ and $L = 0.5$. Find the corresponding value of t.

(d) $\omega = \frac{1}{h}\ln(\frac{L}{L_0} - 1)$. Find L if $\omega = -2.6, L_0 = 16$ and $h = 1.5$.

Solution.

a)

$$f = \frac{uv}{u+v}$$
$$f(u+v) = uv$$
$$fu + fv = uv$$
$$fu = uv - fv$$
$$fu = v(u-f)$$
$$v = \frac{fu}{u-f}$$

Answer: $v = \frac{fu}{u-f}$

b)

$$(3.4)^{2x+3} = 8.5$$
$$\ln((3.4)^{2x+3}) = \ln(8.5)$$
$$(2x+3)\ln 3.4 = \ln 8.5$$
$$2x + 3 = \ln 8.5 - \ln 3.4$$
$$x = \frac{\ln\frac{8.5}{3.4} - 3}{2} = \frac{\ln 2.5 - 3}{2} \approx -1.042$$

Answer: -1.042

c)

$$\theta = Ve^{-\frac{Rt}{L}}$$
$$e^{-\frac{Rt}{L}} = \frac{\theta}{V}$$
$$-\frac{Rt}{L}\ln e = \ln\frac{\theta}{V}$$
$$t = -\frac{L}{R}\ln\frac{\theta}{V} = \frac{L}{R}\ln\frac{V}{\theta} = \frac{0.5}{0.1}\ln\frac{255}{58} = 7.404$$

Answer: 7.404

d)

$$\omega = \frac{1}{h}\ln\left(\frac{L}{L_0} - 1\right)$$

$$\ln\left(\frac{L}{L_0}-1\right)=\omega h$$
$$\frac{L}{L_0}-1=e^{\omega h}$$
$$L=L_0(e^{\omega h}+1)=16(1+e^{-2.6*1.5})=16.32$$

Answer: 16.32

Question 2

(a) Use polynomial long division to determine the quotient when $3x^3-5x^2+10x+4$ is divided by $3x+1$.

(b) Show, by polynomial long division that

$$\frac{x^3-3x^2+12x-5}{x-2}=(x^2-x+10)+\frac{15}{x-2}.$$

Solution.

a)

$$\frac{3x^3-5x^2+10x+4}{3x+1}=$$

$3x^3 - 5x^2 + 10x + 4$	$3x + 1$
$3x^3 + x^2$	$x^2 - 2x + 4$
$-6x^2 + 10x + 4$	
$-6x^2 - 2x$	
$12x + 4$	
$12x + 4$	
0	

$$=(x^2-2x+4)(3x+1)$$

Answer: x^2-2x+4

b)

$$\frac{x^3-3x^2+12x-5}{x-2}=$$

$x^3 - 3x^2 + 12x - 5$	$x - 2$
$x^3 - 2x^2$	$x^2 - x + 10$
$-x^2 + 12x - 5$	
$-x^2 + 2x$	
$10x - 5$	
$10x - 20$	
15	

$$=x^2-x+10+\frac{15}{x-2}$$

Question 3

A ball is thrown down at 72 $km\ h^{-1}$ speed from the top of a building. The building is 125 metres tall. The distance travelled before it reach the ground is as follows,

$$s = u_0 t + \frac{1}{2} g t^2$$

where: u_0 = initial velocity $(m\ s^{-1})$

g = acceleration due to gravity $(10\ m\ s^{-2})$

t = time (s).

(a) Find the time for the ball to drop to a fifth of the height of the buildings.
(b) Find the time for the ball to reach the ground.

Solution.

$$\frac{1}{2} g t^2 + v_0 t - s = 0$$

a) $s = h/5$:

$$\frac{1}{2} g t^2 + v_0 t - \frac{h}{5} = 0$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + \frac{2}{5} hg}}{\text{g}} = 1\ s$$

(we need only positive root, because time always >0)

Answer: 1 s

b) s=h:

$$\frac{1}{2} g t^2 + v_0 t - h = 0$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2hg}}{\text{g}} = 3.385\ s$$

(we need only positive root, because time always >0)

Answer: 3.385 s