Sample: Engineering - Electrical Engineering Problems

P6.34.

We apply a 5-V-rms 20-kHz sinusoid to the input of a first order RC lowpass filter, and the output voltage in steady state is 0.5 V rms. Predict the steady stage rms output voltage after the frequency of the input signal is raised to 150 kHz and the amplitude remains constant.

Solution.

Denote: $f_1 = 20\,\text{kHz}, f_2 = 150\,\text{kHz}, V_0 = 5\,\text{V}$ is the input rms voltage, $V_1 = 0.5\,\text{V}$ is the output rms voltage at a frequency $f_1 = 20\,\text{kHz}$, V_2 is the output rms voltage at a frequency $f_2 = 150\,\text{kHz}$. The magnitude of the transfer function of the filter is:

$$|H(f)| = \frac{1}{\sqrt{1+(f/f_B)^2}},$$

where f_B is the half-power frequency. On the other hand, this magnitude is the ratio of magnitudes of the output and input voltages and it equals the ratio of the output rms and input rms voltages. So we have:

$$|H(f_1)| = \frac{1}{\sqrt{1+(f_1/f_B)^2}} = \frac{V_1}{V_0};\ 1+\frac{f_1^2}{f_B^2} = \frac{V_0^2}{V_1^2}.$$

Find f_B from the expression above:

$$f_B = \frac{V_1 f_1}{\sqrt{V_0^2 - V_1^2}}.$$

Similarly, write the magnitude of the transfer function at the frequency f_2:

$$|H(f_2)| = \frac{1}{\sqrt{1+(f_2/f_B)^2}} = \frac{V_2}{V_0};\ 1+\frac{f_2^2}{f_B^2} = \frac{V_0^2}{V_2^2}.$$

Substitute in this formula the expression for f_B:

$$1+\frac{f_2^2(V_0^2 - V_1^2)}{f_1^2 V_1^2} = \frac{V_0^2}{V_2^2};\ \frac{f_2^2 V_0^2 - V_1^2(f_2^2 - f_1^2)}{f_1^2 V_1^2} = \frac{V_0^2}{V_2^2};$$

$$V_2 = \frac{V_0 V_1 f_1}{\sqrt{f_2^2 V_0^2 - V_1^2(f_2^2 - f_1^2)}} = 0.067\,\text{V rms}.$$

Answer: 0.067 V rms.

P6.81.

Consider the parallel resonant circuit shown in Figure 6.29 on page 317. Determine the L and C values, given $R = 1\,\text{k}\Omega$, $f_0 = 10\,\text{MHz}$, and $B = 500\,\text{kHz}$. If $I = 10^{-3} \angle 0°\,\text{A}$, draw a phasor diagram showing the currents through each of the elements in the circuit at resonance.
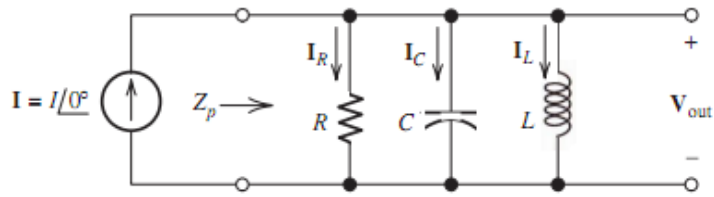
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Figure 6.29 The parallel resonant circuit.

Solution.

The bandwith of the circuit is given by:

$$B = \frac{f_0}{Q_p},$$

where Q_p is the quality factor. Find Q_p:

$$Q_p = \frac{f_0}{B}.$$

On the other hand, the quality factor is given by:

$$Q_p = \frac{R}{2\pi f_0 L}.$$

Considering the two formulas above, we have:

$$\frac{f_0}{B} = \frac{R}{2\pi f_0 L}; \quad L = \frac{BR}{2\pi f_0^2} = 0.796 \ \mu\text{H}.$$

The resonant frequency f_0 and the L and C values are related by the following equation:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

Find the value of the capacitance of the circuit from this equation:

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 f_0^2} \frac{2\pi f_0^2}{BR} = \frac{1}{2\pi BR} = 0.318 \ \text{nF}.$$
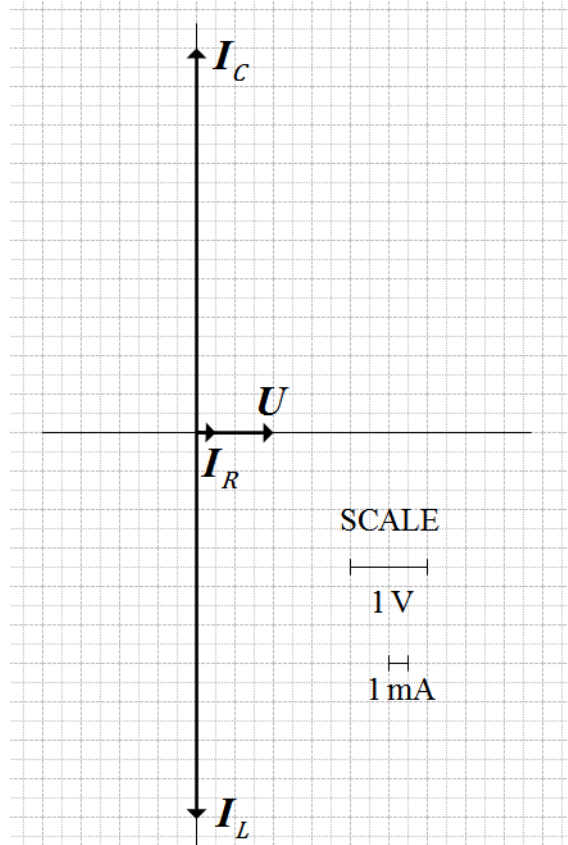
According to Ohm's law find the voltages across each of the elements in the circuit:

$$U_R = U_C = U_L = IR = 1\angle 0° \ \text{V}.$$

According to Ohm's law find the currents through each of the elements in the circuit at resonance:

$$\boldsymbol{I}_R = \frac{U}{R} = 10^{-3}\angle 0° \ \text{A} = 1\angle 0° \ \text{mA}, \quad \boldsymbol{I}_C = j2\pi f_0 C U = 20\angle 90° \ \text{mA}, \quad \boldsymbol{I}_L = \frac{-jU}{2\pi f_0 L} = 20\angle -90° \ \text{mA}.$$

Draw the phasor diagram from the known currents.

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Answer: $L = 0.796\,\mu\text{H},\ C = 0.318\,\text{nF}.$

P6.95. Other combination of R, L, and C have behaviors similar to that of the series resonant circuit. For example, consider the circuit shown in Figure P6.95.

a. Derive an expression for the resonant frequency of this ciruit. (We have defined the resonant frequency to be the frequency for which the impedance is purely resistive).

b. Compute the resonant frequency, given $L = 1\,\text{mH},\ R = 1000\,\Omega,$ and $C = 0.25\,\mu\text{F}.$

c. Use MATLAB to obtain a plot of the impedance magnitude of this circuit for f ranging from 95 to 105 percent of the resonant frequency. Compare the result with that of a series RLC circuit.
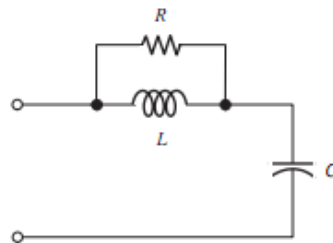

Figure P6.95

Solution.

a. This circuit consists of two parts connected in series. The first part has two elements R and L, connected in parallel, so the impedance of this part is $\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{j\omega L}}=\dfrac{j\omega RL}{R+j\omega L}$. The second part has a single element C, so the impedance of this part is $\dfrac{-j}{\omega C}$. The impedance of the circuit is:

$$Z(w)=\frac{j\omega RL}{R+j\omega L}-\frac{j}{\omega C}.$$

Find the imaginary part of the impedance:

$$\mathrm{Im}[Z(\omega)]=\mathrm{Im}\left[\frac{j\omega RL(R-j\omega L)}{(R+j\omega L)(R-j\omega L)}-\frac{j}{\omega C}\right]=\frac{\omega R^2 L}{R^2+(\omega L)^2}-\frac{1}{\omega C}.$$

The imaginary part of the impedance equals zero at the resonant frequency:

$$\frac{\omega_0 R^2 L}{R^2+(\omega_0 L)^2}-\frac{1}{\omega_0 C}=0;\ \ \omega_0=\frac{R}{\sqrt{L(R^2 C-L)}};\ f_0=\frac{\omega_0}{2\pi}=\frac{R}{2\pi\sqrt{L(R^2 C-L)}}.$$

b. Compute the resonant frequency, given $L=1\,\mathrm{mH}$, $R=1000\,\Omega$, and $C=0.25\,\mu\mathrm{F}$.

$$f_0=\frac{R}{2\pi\sqrt{L(R^2 C-L)}}=10.09\,\mathrm{kHz}.$$

**c.** Write the impedance of the circuit in the following form:

$$Z(f)=\frac{j2\pi fRL}{R+j2\pi fL}-\frac{j}{2\pi fC}.$$

A MATLAB m-file that produces the plot of the impedance magnitude of the circuit is:

```
R=1000; L=1e-3; C=0.25e-6;
%The resonant frequency
f0=R/(2*pi*sqrt(L*(R^2*C-L)));
%The frequency interval
f=[0.95*f0:1e-4*f0:1.05*f0];
%The impedance of the circuit
Z=2*pi*i*f*R*L./(R+2*pi*i*f*L)-i./(2*pi*f*C);
%Display frequency in kHz
f=f./1000;
%Create the plot
plot(f,abs(Z),'k-','linewidth',2);
grid on;
xlabel('Frequency, kHz');
ylabel('Impedance, Ohm')
```
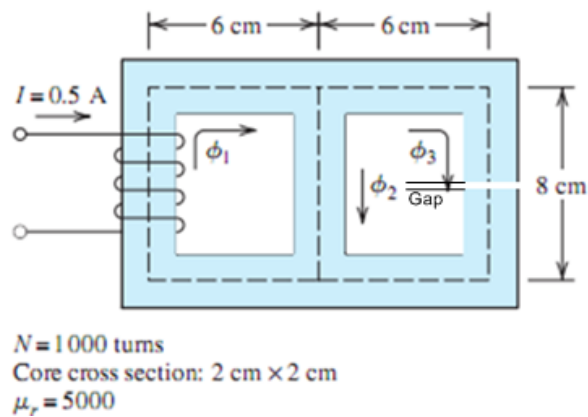
The plot is shown in the figure below.
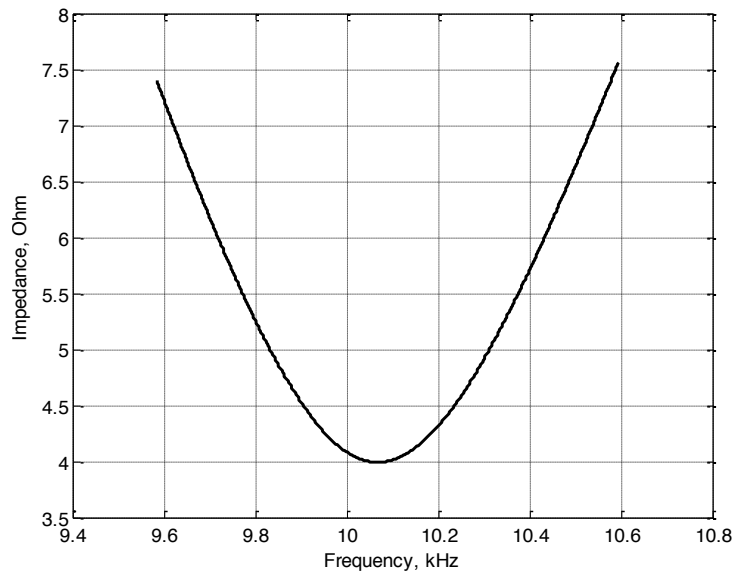
The graph of the circuit impedance near the resonant frequency f_0 behaves as a graph of the input impedance of a series RLC circuit near its resonant frequency.

**Answer:** $f_0 = 10.09 \text{ kHz}$.

P15.30. Repeat problem P15.29 if a gap having a length of 0.5 cm is cut in the right-hand leg of the core. Account for fringing by adding the gap length to each of the cross-sectional dimensions. If you have worked Problem P15.29, prepare a table comparing the fluxes with and without gap. Comment on the results.

$N = 1000$ turns
Core cross section: $2 \text{ cm} \times 2 \text{ cm}$
$\mu_r = 5000$

Solution.

The magnetic circuit is shown in the figure below. First, we find the reluctances of the parts. For the left path, we have:

$$R_1 = \frac{l_1}{\mu_0 \mu_r A_{core}} = \frac{(6+8+6) \times 10^{-2}}{4\pi \times 10^{-7} \times 5000 \times 2 \times 10^{-2} \times 2 \times 10^{-2}} = 7.958 \times 10^4 \text{ A} \cdot \text{turns / Wb}.$$

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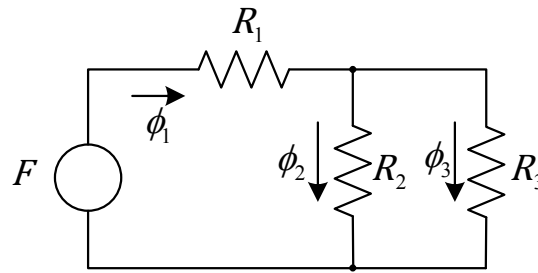
Similarly, the reluctance of the central path is:

$$R_2 = \frac{l_2}{\mu_0 \mu_r A_{core}} = \frac{8 \times 10^{-2}}{4\pi \times 10^{-7} \times 5000 \times 2 \times 10^{-2} \times 2 \times 10^{-2}} = 3.183 \times 10^4 \text{ A} \cdot \text{turns / Wb.}$$

The reluctance of the left path is the sum of the reluctance of the iron core plus the reluctance of a gap. We take fringing into account by adding the gap length to its width and depth in computing area of the gap, so the area of the gap is:

$$A_{gap} = (2+0.5) \times 10^{-2} \times (2+0.5) \times 10^{-2} \text{ m} = 6.25 \times 10^{-4} \text{ m}^2.$$

Then the reluctance of the left part is given by:

$$R_3 = R_{gap} + R_{core} = \frac{l_{gap}}{\mu_0 A_{gap}} + \frac{l_{core}}{\mu_0 \mu_r A_{core}} = \frac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 6.25 \times 10^{-4}} +$$

$$+ \frac{(6+8+6-0.5) \times 10^{-2}}{4\pi \times 10^{-7} \times 5000 \times 2 \times 10^{-2} \times 2 \times 10^{-2}} = 6.366 \times 10^6 + 7.759 \times 10^4 = 6.444 \times 10^6 \text{ A} \cdot \text{turns / Wb.}$$

The magnetomotive force is given by:

$$F = NI = 1000 \times 0.5 = 500 \text{ A} \cdot \text{turns.}$$

The reluctances R_2 and R_3 are connected in parallel. The parallel combination of R_2 and R_3 is connected in series with the reluctance R_1. Reluctances are analogous to resistances. Then the total reluctance is given by:

$$R_{total} = R_1 + \frac{R_2 R}{R_2 + R} = 7.958 \times 10^4 + \frac{3.183 \times 10^4 \times 6.444 \times 10^6}{3.183 \times 10 + 6.444 \times 10^6} = 1.113 \times 10^5 \text{ A} \cdot \text{turns / Wb.}$$

The flux ϕ_1 is:

$$\phi_1 = \frac{F}{R_{total}} = \frac{500}{1.113 \times 10^5} = 4.494 \text{ mWb.}$$

Fluxes are analogous to currents. Thus, we use the current-division principle to determine the fluxes ϕ_2 and ϕ_3:

$$\phi_2 = \phi \frac{R_3}{R_2 + R} = \frac{4.494 \times 10^{-3} \times 6.444 \times 10^6}{3.183 \times 10 + 6.444 \times 10^6} = 4.472 \text{ mWb;}$$

$$\phi_3 = \phi \frac{R_2}{R_2 + R} = \frac{4.494 \times 10^{-3} \times 3.183 \times 10^4}{3.183 \times 10 + 6.444 \times 10^6} = 22.09 \text{ } \mu\text{Wb.}$$

Answer: $\phi_1 = 4.494$ mWb, $\phi_2 = 4.472$ mWb, $\phi_3 = 22.09$ μWb.