Sample: Electromagnetism - Circuits Assignment

Task 1 : **(LO4 :(part 4.3))**

The test equipment requires a supply voltage of 30 volts, however the laboratory into which it is to be installed has a 240 volt ring main supply.
Applying AC theory provide a solution to this problem using a transformer.

For an ideal transformer without load is characterized by the following patterns

$$U_2 = -N_2 \frac{d\Phi}{dt}, U_1 = -N_1 \frac{d\Phi}{dt}$$

$$\frac{U_2}{U_1} = \frac{N_2}{N_1}$$

$$P_1 = I_1 U_1 = P_2 = I_2 U_2$$

Φ -flux of magnetic induction in the core coils

Where U, I, P, N voltage , current, power and amount of turns in the windings . The index indicates the number of the coil, where 1 refers to the primary coil, 2 refers to the secondary coil

In this case, one could say that it is enough to take a proportionate amount of turns in the windings of the transformer to obtain the necessary undervoltage.

But in reality, there are many factors worth to be taken into account, for example, dissipation of energy transmitted from the first coil to the second .

To analyze the operation of the transformer, you can use the linear approximation, or so called equation of linear transformer.

Let i_1, i_2 - instantaneous values of current in the primary and secondary windings, respectively, U_1 - the instantaneous voltage across the primary winding, R_H - load resistance.

$$u_1 = L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt} + i_1 R_1$$

$$L_2 \frac{di_2}{dt} + L_{12} \frac{di_1}{dt} + i_2 R_2 = -i_2 R_H$$

Here L_1, R_1 - inductance and resistance of the primary winding ; L_2, R_2 - inductance and resistance of the secondary winding; L_{12} the mutual inductance of the windings.

We have obtained a system of linear differential equations for the currents in the windings. We can transform these ordinary differential equations in algebraic, if we use the method of complex amplitudes.

For this we consider the response of the system to a sinusoidal signal $u_1 = U_1 e^{-jwt}$

($w = 2\pi f, f -$ frequency, j $-$ imaginary unit) Then $i_1 = I_1 e^{-jwt}$ and so on

$$U_1 = -jwL_1I_1 - jwL_{12}I_2 + I_1R_1$$

$$-jwL_2I_2 - jwL_{12}I_1 + I_2R_2 = -I_2Z_H$$

Where Z_H – complex load resistance

According to this system we can calculate parameters of the transformer. It should be noted that the solution should generally depend on the frequency w .

$$\frac{U_2}{U_1} = \frac{N_2}{N_1} = \frac{240}{30} = 8$$

For the calculation of the parameters of the loaded transformer has a more complex algorithm which is to ensure the validity of the mathematical system

$$U_1 = -jwL_1I_1 - jwL_{12}I_2 + I_1R_1$$

$$-jwL_2I_2 - jwL_{12}I_1 + I_2R_2 = -I_2Z_H$$

Where $|U_2| = |I_2Z_H| = 30\ V, U_1 = 240V$

Task 2 : (LO4 : M1)

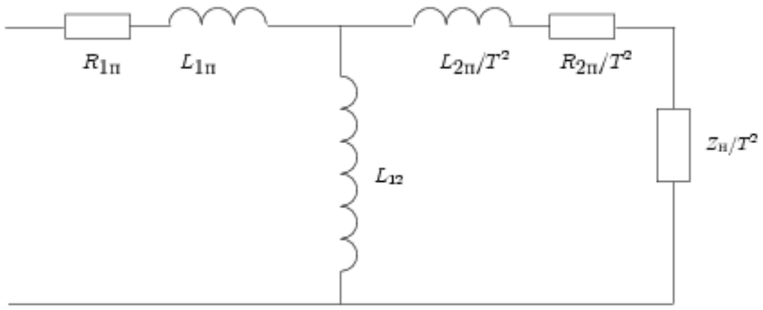
What considerations should be taken into account when designing and installing this transformer?

In applied physics often are preferable empirical equation's rather than theoretically , because it is closer to reality.

There are specific recommendations for the compilation of transformers on the basis of experiments and some theoretical calculations.

Important parameters are price, size, maximum current, the maximum voltage, the efficiency of transformation, the output and input impedance of transformer.

e.g. the transformer primary winding acts like the next scheme



Where T – coefficient of voltage transform; L_{1n}, L_{2n} – inductance of the primary and secondary windings associated with scattering; R_{1n}, R_{2n} – active resistance of the primary and secondary windings, respectively.

Since the present scheme of reactive elements resonance is possible, it is not always acceptable.

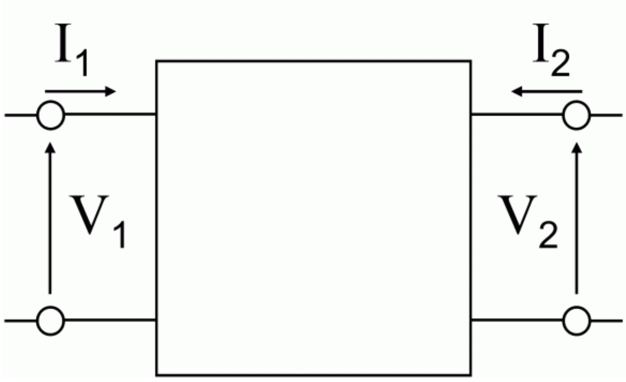
Also not always linear approximation is true. We need to take into account the nonlinearity of the magnetic properties of the core, when the hysteresis phenomenon can be observed.

Task 3 (LO 4: 4.1)

Explain how it might be possible that the 240 volt 50 Hz AC supply voltage could contain harmonics, in particular 3rd and 5th harmonics.
What would these waveforms look like on an oscilloscope, provide a sketch.

Most of the computer and office equipment is a nonlinear electrical load that creates distortions in the supply network. The total effect of these loads is expressed in voltage distortion that affects other equipment receiving power from the same source. This can cause overheating and jitter for other devices, failures in communications and data networks, hardware damage and other undesirable effects.

Consider the behavior of nonlinear loads on the example of non-linear Two-port network Suppose transfer characteristic is known as a polynomial series.



$$V_2 = a_0 + a_1 * V_1 + a_2 {V_1}^2 + a_3 {V_1}^3 + \cdots$$

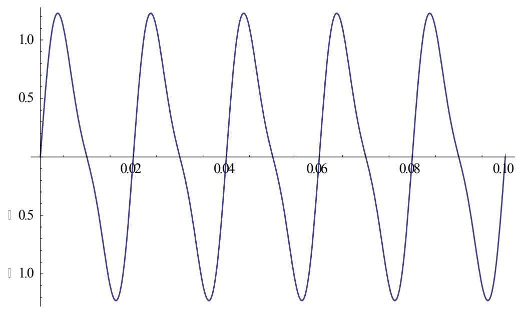
Then, if $V_1 = V_0 Sin(wt)$

$$V_2 = a_0 + a_1 * V_0 * Sin(wt) + a_2 {V_0}^2 * (Sin(wt)^2) + a_3 {V_0}^3 * (Sin(wt)^3) + \cdots$$

and because the $Sin(wt)^2 = \frac{1-Cos(2wt)}{2}, Sin(wt)^3 = \frac{3Sin(wt)-Sin(3wt)}{4}$ the output voltage harmonics will be present.

By the way , some parts of scheme may play the role of filters and amplify some harmonics (for example RC filters)

And this distortion signal will look like follows:



Task 4 : (LO4 : 4.2)

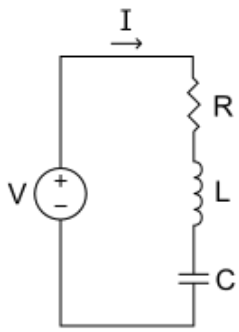
The characteristics of the test equipment, as seen by the supply, have been given to you in terms of its R, L and C properties, as follows:

Resistance 10 Ohms, Inductance 1.126 Henry and capacitance 1 micro Farad.

At what frequency would this equipment resonate?

There are a several characteristic component connections - series, parallel or others.. Consider a series rlc resonant circuit.

According to Kirchhoff's second rule:



$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V(t)$$

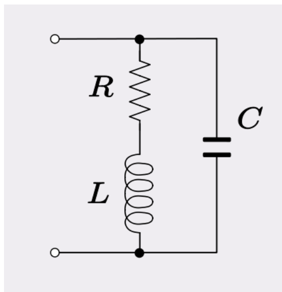
free oscillation → $V(t) = 0$

solution will look like $q(t) = A_0 e^{-Rt/2L} \sin(wt + a_0)$, w=$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

Where w is desired frequency

In this case $\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\frac{1}{1.126H*10^{-6}F} - \frac{10^2 Om^2}{4(1.126H)^2}} \approx 942Hz$

In the same way may be obtain frequency for another scheme



W'=$\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{1.126H*10^{-6}F} - \frac{10^2 Om^2}{(1.126H)^2}} \approx 942Hz$

So all this scheme will resonate at the frequency close to 942 Hz