Sample: Optics - Biomedical Physics Homework

1. A metal surface has a photoelectric cutoff wavelength of 325.6 nm. It is illuminated with light of wavelength 259.8 nm. What is the maximum kinetic energy of the photoelectrons?

Solution:
Given:
λ = 259.8 nm,
λ_c = 325.6 nm.

Einstein's formula relates the maximum kinetic energy (K_{max}) of the photoelectrons to the frequency of the absorbed photons (f) and the threshold frequency (f_c) of the photoemissive surface.

$$K_{max} = h(f - f_c)$$

The frequency is

$$f = \frac{c}{\lambda}$$

Where $c = 3 \cdot 10^8$ m/s is speed of light, and
$h = 6.63 \cdot 10^{-34}$ J s $= 4.14 \cdot 10^{-15}$ eV·s is Planck's Constant

Thus,

$$K_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_c}\right)$$

$$K_{max} = 6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8 \left(\frac{1}{259.8} - \frac{1}{325.6}\right)\frac{1}{10^{-9}} = 1.55 \cdot 10^{-19}\ \mathrm{J}$$

$$K_{max} = 4.14 \cdot 10^{-15} \cdot 3 \cdot 10^8 \left(\frac{1}{259.8} - \frac{1}{325.6}\right)\frac{1}{10^{-9}} = 0.97\ \mathrm{eV}$$

Answer. $K_{max} = 1.55 \cdot 10^{-19}\ \mathrm{J} = 0.97\ \mathrm{eV}$

2. A certain cavity has a temperature of 1150 K. Assume the blackbody radiation.

(a) At what wavelength will the radiancy have its maximum value?

(b) What is the ratio between the radiancy at twice the wavelength found in part (a) and the maximum radiancy?

(Note: the radiancy is defined by R(λ)=(c/4)u′(λ), with u′(λ) the energy density.)

Solution:

(a)

The experiments show that the maximum wavelength is inversely proportional to the temperature. In fact, we have found that if you multiply λ_{max} and the temperature, you obtain a constant, in what is known as Wein's displacement law:

$$\lambda_{max}T = 2.898\cdot 10^{-3}\ \mathrm{m\cdot K}$$

Thus,

$$\lambda_{max} = \frac{2.898\cdot 10^{-3}}{T} = \frac{2.898\cdot 10^{-3}}{1150} = 2.52\ \cdot 10^{-6}\ \mathrm{m} = \ 2.52\ \mathrm{\mu m}$$

(b)

The radiance is related to the energy density (energy per unit volume) u′(λ) in the relationship

$$R(\lambda) = \frac{c}{4}u'(\lambda)$$

This is obtained by determining the amount of radiation passing through an element of surface area within the cavity.

Planck generated a theoretical expression for the wavelength distribution (radiance)

$$R(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

where c = 3·10^8 m/s is speed of light, and

h = 6.63 · 10^{-34} J s = 4.14 · 10^{-15} eV·s is Planck's Constant

k = 1.3807·10 $^{-23}$ joule s per kelvin (J · K $^{-1}$) is Boltzmann's constant.

For $\lambda = 2\lambda_{max} = 5.04\cdot 10^{-6}$ m we obtain

$$\frac{R(\lambda)}{R(\lambda_{max})} = \frac{R(2\lambda_{max})}{R(\lambda_{max})} = \left(\frac{\lambda_{max}}{2\lambda_{max}}\right)^5 \frac{e^{\frac{hc}{\lambda_{max}kT}} - 1}{e^{\frac{hc}{2\lambda_{max}kT}} - 1}$$

$$\frac{R(\lambda)}{R(\lambda_{max})} = \frac{R(2\lambda_{max})}{R(\lambda_{max})} = \left(\frac{1}{2}\right)^5 \frac{e^{\frac{6.63\cdot 10^{-34}\cdot 3\cdot 10^8}{2.52\cdot 10^{-6}\cdot 1.38\cdot 10^{-23}\cdot 1150}} - 1}{e^{\frac{6.63\cdot 10^{-34}\cdot 3\cdot 10^8}{5.04\cdot 10^{-6}\cdot 1.38\cdot 10^{-23}\cdot 1150}} - 1} = 0.407$$

Answer. a) $\lambda_{max} = 2.52\ \mathrm{\mu m}$, b) $\frac{R(\lambda)}{R(\lambda_{max})} = 0.407$