Sample: Astronomy Astrophysics - Astronomy Planets Calculations

1. If Jupiter has an average temperature of 125 K, at what wavelength is it emitting the most energy according to Wien`s Law?

Solution

We know the Wien`s Law $\lambda_{\max} T = b$, where $\lambda_{\max}$ is the peak wavelength, T is temperature of object, $b = 2.898 \cdot 10^{-3}\ \mathrm{m \cdot K}$. From hence, if Jupiter has temperature $T = 125K$,

$$\lambda_{\max} = \frac{b}{T} = \frac{2.898 \cdot 10^{-3}\ \mathrm{m \cdot K}}{125\mathrm{K}} = 0.0232 \cdot 10^{-3} m = 2.32 \cdot 10^{-5} m$$

Answer:

$$\lambda_{\max} = 2.32 \cdot 10^{-5} m$$

2. If a new planet is discovered beyond the orbit Neptune which has a radius of three times the Earth`s radius and a density of 1200 kg per meter cubed, what is the mass of the new planet in terms of Earth`s mass?

Solution

We know that the mass of planet which have radius R and density ρ is

$$M = \frac{4\pi}{3} \rho R^3.$$

The radius of Earth is $R_e = 6371km = 6.371 \cdot 10^6\ m$, mass of Earth is $M_e = 5.9742 \cdot 10^{24}\ kg$.

Radius of new planet is $R = 3R_e = 1.9113 \cdot 10^7\ m$, mass of new planet is

$$M = \frac{4 \cdot 3.1416}{3} \cdot 1200 \frac{kg}{m^3} \left(1.9113 \cdot 10^7 m\right)^3 = 3.5096 \cdot 10^{25} kg = 5.876 M_e$$

Answer:

$$M = 5.876 M_e$$

3. If Phobos has a semi-major axis of 19000 km and Mars has a mass of 0.11 M_e (of Earth), what is its period around Mars in hours? Be careful with your units here.

Solution

We know the third Kepler's law $T^2 = \frac{4\pi^2 a^3}{GM_{Mars}}$, where T is orbital period of satellite, a is semi-major axis of its orbit, $M_{Mars} = 0.11M_E$ is mass of the Mars (mass of satellite (Phobos) is very small comparing to mass of the Mars), $G = 6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$ is gravitational constant

From hence we have

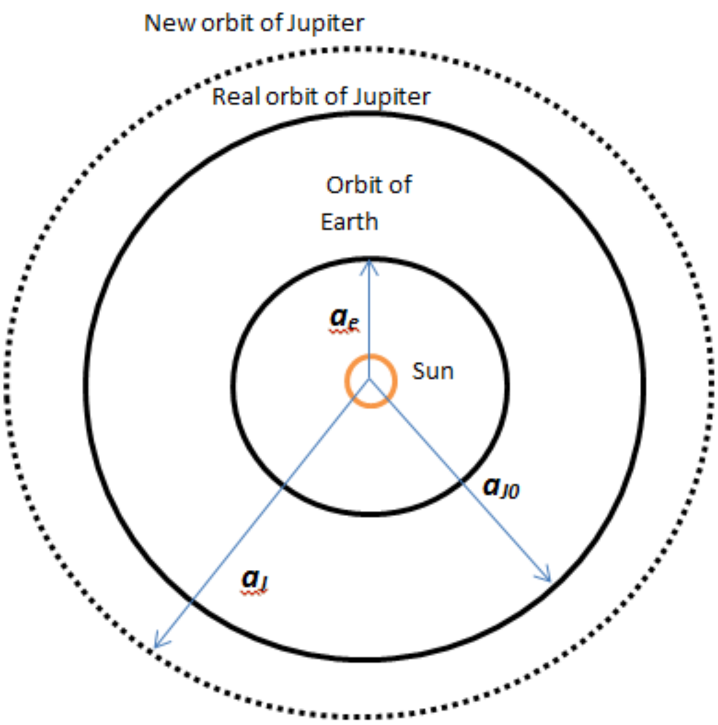
$$M = 6.5716 \cdot 10^{23} kg$$
$$a = 19000000m$$
$$T = 78600\text{sec} = 21.83 hours$$

Answer:

$$T = 78600\text{sec} = 1310\text{min} = 21.83 hours$$

4. If Jupiter were really 7.52 AU from the Sun instead of 5.20 AU, what would the affect be on the energy it gets from the Sun? What would be the effect on its appearance (size and brightness) from Earth?

Solution



The energy(which is emitted from sun) is uniformly distributed on surface of sphere centered in the sun. From hence, if Jupiter has semi-major axis $a_J = 7.52 a.u.$ instead $a_{J0} = 5.20 a.u.$, it gets energy from sun $E_J = E_{J0}\frac{4\pi a_{J0}^2}{4\pi a_J^2} = E_{J0}\frac{a_{J0}^2}{a_J^2} = 0.478E_{J0}$ instead initial E_{J0}.

We have, that the observation appearance of Jupiter depends on distance between Jupiter and Earth. It changes between $R_O = a_J - a_e = (7.52-1)a.u. = 6.52 a.u.$ (in opposition) and $R_c = a_J + a_e = (7.52+1)a.u. = 8.52 a.u.$ (in conjunction) (a_e is radius of orbit of Earth).

Initially distance changes between $R_{O0} = a_{J0} - a_e = (5.20-1)a.u. = 4.20 a.u.$ and $R_{C0} = a_{J0} + a_e = (5.20+1)a.u. = 6.20 a.u.$

From hence, we get that the angular size of Jupiter in opposition will be smaller in $\frac{R_O}{R_{O0}} = \frac{6.52}{4.20} = 1.55$ times (Jupiters linear radius is very small comparing to the distance between Earth and Jupiter, from hence it's angular radius is proportional to distance between Jupiter and Earth), in conjunction it will be smaller in $\frac{R_C}{R_{C0}} = \frac{8.52}{6.2} = 1.37$ times .

Brightness of Jupiter is proportional to part of solar energy, which is scattered by Jupiter (this part is proportional to energy which it gets from sun). Brightness is inversely as the square of distance between Earth and Jupiter.

From hence, in opposition we get Jupiter's brightness B_O in comparison with initial value in opposition B_{O0} will be smaller as $\frac{B_O}{B_{O0}} = \frac{E_J}{E_{J0}}\left(\frac{R_O}{R_{O0}}\right)^{-2} = 0.199$, in conjunction Jupiter's brightness B_C in comparison with initial value in conjunction B_{C0} will be $\frac{B_C}{B_{C0}} = \frac{E_J}{E_{J0}}\left(\frac{R_C}{R_{C0}}\right)^{-2} = 0.349$.

Answer: $E_J = 0.478E_{J0}$

In opposition it's angular size will be in 1.55 times smaller, in conjunction in 1.37 times smaller.

In opposition it's brightness changes
in comparison with initial value as

$$\frac{B_O}{B_{O0}} = 0.199$$

In conjunction it's brightness changes
in comparison with initial value

$$\frac{B_C}{B_{C0}} = 0.349$$