Sample: Quantum Mechanics - Physics Assignment

1. The normalization condition is $\int |\psi(x)|^2 dx = 1$. The wave-function for $-\pi < x < \pi$ might be rewritten as $\psi(x) = A(e^{ix} + e^{-ix}) = 2A\cos x$. Thus, according to normalization condition,
$4A^2 \int_{-\pi}^{\pi} \cos^2 x\, dx = 4A^2 \int_{-\pi}^{\pi} \frac{1+\cos 2x}{2} dx = 4A^2 \left(\frac{x}{2} + \frac{\sin 2x}{4}\right)\Big|_{-\pi}^{\pi} = 4A^2 \cdot \pi = 1$. Hence, $A = \frac{1}{2\sqrt{\pi}}$ and $\psi(x) = \cos\frac{x}{\sqrt{\pi}}$ for $-\pi < x < \pi$.

$$P\left(0 < x < \frac{\pi}{8}\right) = \int_0^{\frac{\pi}{8}} |\psi(x)|^2 dx = \frac{1}{\pi}\int_0^{\frac{\pi}{8}} \cos^2 x\, dx = \frac{1}{\pi}\left(\frac{x}{2} + \frac{\sin 2x}{4}\right)\Big|_0^{\frac{\pi}{8}} = \frac{1}{\pi}\left(\frac{\pi}{16} + \frac{1}{4\sqrt{2}}\right) .$$

2. Plugging in $\Psi(x,t) = \psi(x) f(t)$ into time-dependent Schroedinger equation yields
$i\hbar\psi f_t = \frac{-\hbar^2}{2m}\psi_{xx} f(t) + V(x)\psi(x) f(t)$. Dividing the last equation by $\psi(x) f(t)$, obtain
$\frac{i\hbar f_t}{f} = \frac{\left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x)}{\psi(x)}$. Since the right side does not depend on t explicitly, and left side is function of t , then $\frac{i\hbar f_t}{f} = \frac{\left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x)}{\psi(x)} = E = const$, which yields time-independent Schrodinger equation $\frac{-\hbar^2}{2m}\psi_{xx} + V(x)\psi(x) = E\psi(x)$.

3.

a) Since for certain n , $l = 0..n-1$, thus for $n = 6$, $l = 0..5$.
b) Since for certain l , $m_l = -l..l$, for $l = 6$, $m_l = -6..6$.
c) Knowing that for certain n , $l = 0..n-1$, the smallest possible value of n , for which $l = 4$ is $n = 5$.
d) Knowing that for certain l , $m_l = -l..l$, the smallest possible value of l , for which $m_l = 4$ is $l = 4$.

4.

a)

$$\langle r \rangle = \int_0^\infty r^2 |R(r)|^2 dr = \frac{4}{a_0^3}\int_0^\infty r^3 e^{\frac{-2r}{a_0}} dr = \left[t = 2\frac{r}{a_0}; dt = \frac{2}{a_0}dr\right] = \frac{1}{4}\frac{a_0^4}{a_0^3}\int_0^\infty t^3 e^{-t} dt = \frac{a_0}{4}\Gamma(4) = \frac{a_0}{4}\cdot 3! = \frac{3}{2}a_0 .$$

b) The Coulomb potential is $U = \frac{-\alpha}{r}$. Thus,

$$\langle U \rangle = -4\frac{\alpha}{a_0^3}\int_0^\infty \frac{r^2}{r} e^{\frac{-2r}{a_0}} dr = \left[t = 2\frac{r}{a_0}; dt = \frac{2}{a_0}dr\right] = -4\frac{\alpha}{a_0^3}\left(\frac{a_0}{2}\right)^2 \int_0^\infty t e^{-t} dt = -4\frac{\alpha}{4a_0}\Gamma(2) = \frac{-\alpha}{a_0} .$$