Sample: Algorithms Quantitative Methods - Math Assignment

Question #1

(a) Let take the number of calls in day time is:

Category	Number of calls in a day time
Married Woman	x_{11}
Married Man	x_{21}
Single Woman	x_{31}
Single Man	x_{41}

And the number of calls in the evening time is:

Category	Number of calls in an evening time
Married Woman	x_{12}
Married Man	x_{22}
Single Woman	x_{32}
Single Man	x_{42}

In this case we know that the total number of the phone calls (we will pay $1 per every call) is:

$$C_{call} = \sum x_{ij} = x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} + x_{41} + x_{42}.$$

As we know that only 50% of calls during a day and 88% of calls during evening are picked up and only 50% of the people who answer the calls agree to participate in the survey. For every refuse we should also pay $1 and “refused” costs will be:

$$C_{refused} = 0.5 \cdot 0.5 \cdot (x_{11} + x_{21} + x_{31} + x_{31}) + 0.5 \cdot 0.88 \cdot (x_{12} + x_{22} + x_{32} + x_{42}).$$

We know that if the person who answers the phone agrees to participate in the survey, it costs $10 to complete the survey during day time and $20 during evening time (due to the higher labour cost) and only 50% of the people who answer the calls agree to participate in the survey. In this case the costs for survey will be equal to:

$$C_{answered} = 10 \cdot 0.5 \cdot 0.5 \cdot (x_{11} + x_{21} + x_{31} + x_{31}) + 20 \cdot 0.5 \cdot 0.88 \cdot (x_{12} + x_{22} + x_{32} + x_{42}).$$

Total costs will be equal:

$$C_{calls} + C_{refused} + C_{answered} = x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} + x_{41} + x_{42} + 0.5 \cdot 0.5 \cdot (x_{11} + x_{21} + x_{31} + x_{31}) +$$
$$+ 0.5 \cdot 0.88 \cdot (x_{12} + x_{22} + x_{32} + x_{42}) + 10 \cdot 0.5 \cdot 0.5 \cdot (x_{11} + x_{21} + x_{31} + x_{31}) +$$
$$+ 20 \cdot 0.5 \cdot 0.88 \cdot (x_{12} + x_{22} + x_{32} + x_{42}) =$$
$$= 3.75 \cdot (x_{11} + x_{21} + x_{31} + x_{31}) + 10.24 \cdot (x_{12} + x_{22} + x_{32} + x_{42}).$$

We obtained that the total cost to complete the survey, which we need to minimize is:

$$C = 3.75 \cdot (x_{11} + x_{21} + x_{31} + x_{31}) + 10.24 \cdot (x_{12} + x_{22} + x_{32} + x_{42}) \to \min .$$

Now we will find the constraints:

The project requires responses from:

- At least 120 married women: from this category only 10% during the day time and 14% during the evening agrees to participate in the survey:

$$0.1 \cdot x_{11} + 0.14 \cdot x_{12} \ge 120 .$$

- At least 150 married men: from this category only 5% during the day time and 14% during the evening agrees to participate in the survey: $0.05 \cdot x_{21} + 0.14 \cdot x_{22} \ge 150$.
- At least 110 single women: from this category only 5% during the day time and 8% during the evening agrees to participate in the survey: $0.05 \cdot x_{31} + 0.08 \cdot x_{32} \ge 110$.
- At least 100 single men: from this category only 5% during the day time and 8% during the evening agrees to participate in the survey: $0.05 \cdot x_{41} + 0.08 \cdot x_{42} \ge 100$.

And we also know that due to the staff hiring policy, at most half of all phone calls can be made during evening, or in other words the evening calls at most a half of all phone calls:

$$x_{12} + x_{22} + x_{32} + x_{42} \le \frac{1}{2}(x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} + x_{41} + x_{42}) \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}(x_{11} - x_{12} + x_{21} - x_{22} + x_{31} - x_{32} + x_{41} - x_{42}) \ge 0.$$

Linear programming problem:

The objective function (we should minimized the cost):

$$C = 3.75 \cdot (x_{11} + x_{21} + x_{31} + x_{31}) + 10.24 \cdot (x_{12} + x_{22} + x_{32} + x_{42}) \to \min$$

Constraints:

1) $0.1 \cdot x_{11} + 0.14 \cdot x_{12} \ge 120$.

2) $0.05 \cdot x_{21} + 0.14 \cdot x_{22} \ge 150$.

3) $0.05 \cdot x_{31} + 0.08 \cdot x_{32} \ge 110$.

4) $0.05 \cdot x_{41} + 0.08 \cdot x_{42} \ge 100$.

5) $\frac{1}{2}(x_{11} - x_{12} + x_{21} - x_{22} + x_{31} - x_{32} + x_{41} - x_{42}) \ge 0.$

We solved this problem with the help of MS Excel:

Number of calls	Day time	Evening time
Married Women	1200	0
Married Men	0	1072

Single Women	2200	0
Single Men	2000	0

Number of Responses	% during day time	% during evening			
A Married Woman	10%	14%	120	>=	120
A Married Man	5%	14%	150	>=	150
A Single Woman	5%	8%	110	>=	110
A Single Man	5%	8%	100	>=	100

At most half of all phone calls can be made during evening:	2164	>=	0

Costs	
$1 per every call	6472
Picked up and refused	1821,68
Participeted	22933,6
Total	31227,28

According to the Excel solution we should make 1200 phone calls to the married woman during a day time; 1072 phone calls to the married man durring evening time; 2200 phone calls to the single woman during a day time; 2000 phone calls to a single man during a day time. And in this case we will complete our survey (we will have answers from 120 married woman, 150 married man, 110 single woman and 100 single man) and the cost of the survey equal $31227.28.

Answer: we should make 1200 phone calls to the married woman during a day time; 1072 phone calls to the married man durring evening time; 2200 phone calls to the single woman during a day time; 2000 phone calls to a single man during a day time. And in this case we will complete our survey (we will have answers from 120 married woman, 150 married man, 110 single woman and 100 single man) and the cost of the survey equal $31227.28.

(b) Let take that the total number of the newspaper ads is x and the total number of the TV commercials is y.

We know that it costs $2,000 per newspaper ads and $15,000 per TV commercials and we know that thw annual marketing budger is $200,000. If the total number of the newspaper ads represented by x and the total number of the TV commercials represented by y, we will have the constraint for the budget:

$$2000\cdot x+15000\cdot y\le 200000.$$

We know also that at most 50 newspaper ads and 12 TV commecials can be placed during a year:

$$x\le 50,$$

$$y\le 12.$$

The number of new customers can be reached by each advertisement is shown in the table below:

Number of ads	Customers reached per ad
Newspaper ads up to 10 times	1000
Newspaper ads from 11 to 20 times	600
Newspaper ads above 21 times	200
TV commercials up to 5 times (max is 4)	10,000
TV commercials from 5 to 10 times	6,000
TV commercials above 11 times	2,000

According to this table we have next situation for the objective function (we should maximise the number of customers reached by ads):

1) From the newspaper ads:

- if the number of the newspaper ads is less or equal 10, then:

$$x\le 10:$$
$$C_{newspaper}=1000\cdot x.$$

- if the number of the newspaper ads is more or equal 11, but less or equal to 20:

$$11\le x\le 20:$$
$$C_{newspaper}=1000\cdot 10+600\cdot(x-10)=600\cdot x+4000.$$

- if the number of the newspaper ads is more or equal to 21:

$$x\ge 21:$$
$$C_{newspaper}=1000\cdot 10+600\cdot 10+200\cdot(x-20)=200\cdot x+12000.$$

2) For the TV commercials:

- if the number of the TV commercials is less than 5, then:

$$y\le 5:$$
$$C_{TV}=10000\cdot y.$$

- if the number of the TV commercials is more or equal 5, but less or equal to 10:

$5 \le y \le 10:$

$C_{TV} = 10000 \cdot 5 + 6000 \cdot (y - 5) = 6000 \cdot y + 20000.$

- if the number of the TV commercials is more or equal to 11:

$y \ge 11:$

$C_{TV} = 10000 \cdot 5 + 6000 \cdot 5 + 2000 \cdot (y - 10) = 2000 \cdot y + 60000.$

Linear programming problem:

The objective function (we should minimized the cost):

$$C = \begin{cases} 1000 \cdot x + 10000 \cdot y, x \le 10, y \le 5 \\ 1000 \cdot x + 6000 \cdot y + 20000, x \le 10, 5 \le y \le 10 \\ 1000 \cdot x + 2000 \cdot y + 60000, x \le 10, y \ge 11 \\ 600 \cdot x + 4000 + 10000 \cdot y, 11 \le x \le 20, y \le 5 \\ 600 \cdot x + 6000 \cdot y + 24000, 11 \le x \le 20, 5 \le y \le 10 \to \max \\ 600 \cdot x + 2000 \cdot y + 64000, 11 \le x \le 20, y \ge 11 \\ 200 \cdot x + 12000 + 10000 \cdot y, x \ge 21, y \le 5 \\ 200 \cdot x + 6000 \cdot y + 32000, x \ge 21, 5 \le y \le 10 \\ 200 \cdot x + 2000 \cdot y + 72000, x \ge 21, y \ge 11 \end{cases}$$

The constraints:

1) $2000 \cdot x + 15000 \cdot y \le 200000$.

2) $x \le 50$.

3) $y \le 12$.

We solved this problem with the help of MS Excel:

	Number
Newspaper ads	25
TV ads	10

	x	y				
Cost	2000	15000	=	200000	<=	200000
Maximum number of Newspaper ads	1	0	=	25	<=	50
Maximum number of TV ads	0	1	=	10	<=	12

Number of customers reached by ads	
Newspaper ads	17000
TV ads	80000
Total	97000

According to the Excel solution to maximise the number of customers reached by ads,Tim should make 25 newspaper ads and 10 TV commercials. In this case the number of customers reached by ads will be equal to 97,000.

Answer: to maximise the number of customers reached by ads,Tim should make 25 newspaper ads and 10 TV commercials. In this case the number of customers reached by ads will be equal to 97,000.