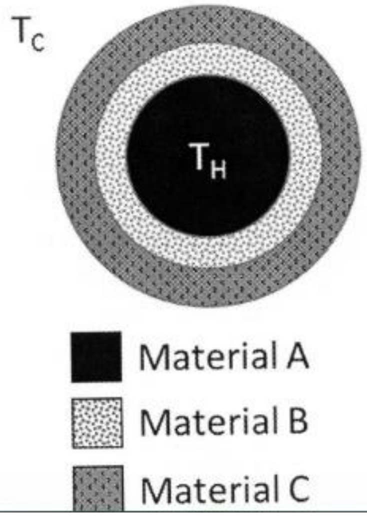
Sample: Molecular Physics Thermodynamics - Heat Trasnfer

4. A particular nuclear fuel rod design consists of a central fissile core (material A), a neutron moderator (material B), and an outer structural and heat transfer cladding (material C). The fuel rods are bundled and bathed in a heat transfer fluid (pressurized water) at temperature T_C (also a neutron moderator). If the fuel rod is heated too much, the neutron moderator can crack increasing the amount of neutrons released from a fuel rod. The increased neutron flux from the damaged rod interacting with the neighboring fuel rods increases the heat output in a chain reaction and may lead to a runaway reactor. The moderator is damaged when the thermal gradient of the moderator exceeds 5260 K/cm.

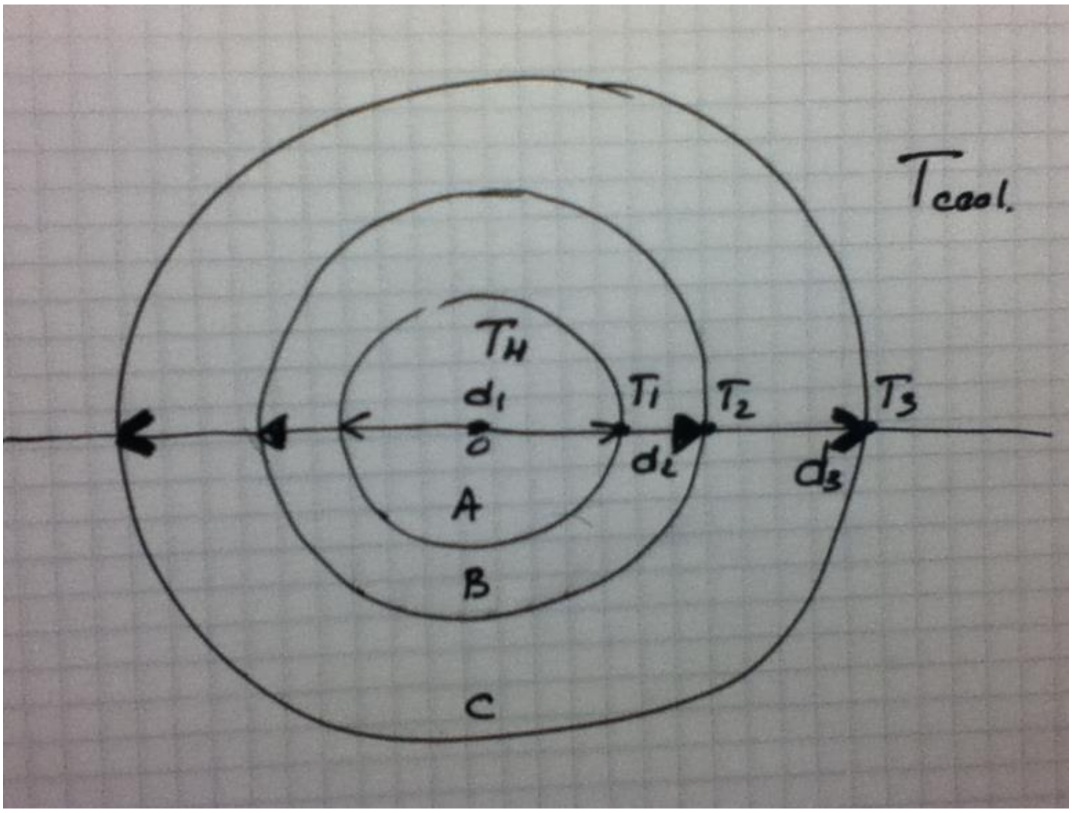
Physical properties and dimensions of system are: k_A=13.5 W/m K; k_B= 81.7 W/m K; k_C= 139.4 W/m K; h= $5.36\text{x}10^5$ W/m² K. Diameter of material A is ¼ inch, thickness of material B is $^1/_{32}$ inch, and thickness of material C is $^1/_{16}$ inch.

(A) Given the following diagram and information, calculate T_H for the minimum operational temperature at which the moderator will crack with a cooling fluid temperature of T_C=1986K. Assume that heat transfer within the rod occurs solely by conduction and that the heat generated in the fissile core occurs at the outer edge of material A. (20 pts)

(B) Thermolysis (water breaks down to hydrogen and oxygen) occurs at 2500°C making cooling of the reactor impossible. At what T_H does this occur? (20 pts)



Solution:



Given

$$d_1 = 0.00635\ m$$

$$d_2 = 0.0079373\ m$$

$$d_3 = 0.0111126\ m$$

$$k_A = 13.5\ \frac{W}{m}$$

$$k_B = 81.7\ \frac{W}{m}$$

$$k_C = 139.4\ \frac{W}{m}$$

$$T_{cool} = 1986\ K$$

$$alpha = h = 5.36 \cdot 10^5\ \frac{W}{m^2K}$$

$$CriticalGradient = 5260\ \frac{K}{cm} = 526000\ \frac{K}{m}$$

Write the equations for the linear heat flux

$$q_l = alpha \cdot (T_3 - T_{cool})\pi \frac{d_3}{2}$$

$$q_l = 2\pi k_B \frac{T_1 - T_2}{ln\left(\frac{d_2}{d_1}\right)}$$

$$q_l = 2\pi k_c \frac{T_2 - T_3}{ln\left(\frac{d_3}{d_2}\right)}$$

Write the equation for $CriticalGradient$

$$CriticalGradient = \frac{T_1 - T_2}{\frac{d_2 - d_1}{2}} = 526000$$

I will do the conversion for the flow in which the possibility of an accident

$$q_l = 2\pi k_B \frac{T_1 - T_2}{ln\left(\frac{d_2}{d_1}\right)} = 2\pi k_B \frac{\frac{T_1 - T_2}{\frac{d_2 - d_1}{2}}}{ln\left(\frac{d_2}{d_1}\right)} \cdot \frac{d_2 - d_1}{2} = 2\pi k_B \frac{CriticalGradient}{ln\left(\frac{d_2}{d_1}\right)} \cdot \frac{d_2 - d_1}{2}$$

So now we know $q_l = q_l(accident)$

Now we need to find T_3

$$q_l = alpha \cdot (T_3 - T_{cool})\pi \frac{d_3}{2} = 2\pi k_B \frac{CriticalGradient}{ln\left(\frac{d_2}{d_1}\right)} \cdot \frac{d_2 - d_1}{2}$$

So

$$T_3 = T_{cool} + \left(2\pi k_B \frac{CriticalGradient}{ln\left(\frac{d_2}{d_1}\right)} \cdot \frac{d_2 - d_1}{2}\right) \cdot \frac{1}{alpha \cdot \pi \frac{d_3}{2}}$$

After we need to find T_2

$$2\pi k_c \frac{T_2 - T_3}{ln\left(\frac{d_3}{d_2}\right)} = 2\pi k_B \frac{CriticalGradient}{ln\left(\frac{d_2}{d_1}\right)} \cdot \frac{d_2 - d_1}{2}$$

$$T_2 = T_3 + \left(2\pi k_B \frac{CriticalGradient}{ln\left(\frac{d_2}{d_1}\right)} \cdot \frac{d_2 - d_1}{2}\right) \cdot \frac{ln\left(\frac{d_3}{d_2}\right)}{2\pi k_c}$$

And after we need to find T_1 from equation

$$CriticalGradient = \frac{T_1 - T_2}{\frac{d_2 - d_1}{2}} = 526000$$

$$T_1 = T_2 + 526000 \cdot \left(\frac{d_2 - d_1}{2}\right)$$

Calculation in MAPLE

> $d1 := 6.35 \cdot 10^{-3}$

$$d1 := 0.006350000000$$

> $d2 := d1 + 2 \cdot 7.938 \cdot 10^{-4}$

$$d2 := 0.007937600000$$

> $d3 := d2 + 2 \cdot 1.5875 \cdot 10^{-3}$

$$d3 := 0.01111260000$$

> $Tc := 1986$

$$Tc := 1986$$

> $\text{alpha} := 5.36 \cdot 10^{5}$

$$\alpha := 5.3600000\ 10^{5}$$

> $ka := 13.5$

$$ka := 13.5$$

> $kb := 81.7$

$$kb := 81.7$$

> $kc := 139.4$

$$kc := 139.4$$

> $CriticalGradientTemp := 526000$

$$CriticalGradientTemp := 526000$$

> $qldamage := \dfrac{2 \cdot \text{Pi} \cdot kb \cdot CriticalGradientTemp \cdot 0.5 \cdot (d2 - d1)}{\ln\left(\dfrac{d2}{d1}\right)}$

$$qldamage := 3.057313905\ 10^{5}\ \pi$$

> $T3 := Tc + \dfrac{qldamage \cdot 2}{\text{alpha} \cdot \text{Pi} \cdot d3}$

$$T3 := 2088.657233$$

> $T2 := T3 + \dfrac{qldamage \cdot \ln\left(\dfrac{d3}{d2}\right)}{2 \cdot \text{Pi} \cdot kc}$

$$T2 := 2457.627908$$

> $T1 := T2 + CriticalGradientTemp \cdot \dfrac{(d2 - d1)}{2}$

$$T1 := 2875.166708$$

So

$$T_1 = 2875\ K$$

$$T_2 = 2458\ K$$

$$T_3 = 2089\ K$$

Solution a -> T(H)=2875 K

Solution Task (B)

If $T_{cool} = 2500\ C = 2773\ K\ then\ recalculate$

> > $d1 := 6.35 \cdot 10^{-3}$

$$d1 := 0.006350000000$$

> $d2 := d1 + 2 \cdot 7.938 \cdot 10^{-4}$

$$d2 := 0.007937600000$$

> $d3 := d2 + 2 \cdot 1.5875 \cdot 10^{-3}$

$$d3 := 0.01111260000$$

> $Tc := 2500 + 273$

$$Tc := 2773$$

> $\text{alpha} := 5.36 \cdot 10^5$

$$\alpha := 5.3600000\ 10^5$$

> $ka := 13.5$

$$ka := 13.5$$

> $kb := 81.7$

$$kb := 81.7$$

> $kc := 139.4$

$$kc := 139.4$$

> $CriticalGradientTemp := 526000$

$$CriticalGradientTemp := 526000$$

> $qldamage := \dfrac{2 \cdot \text{Pi} \cdot kb \cdot CriticalGradientTemp \cdot 0.5 \cdot (d2 - d1)}{\ln\left(\frac{d2}{d1}\right)}$

$$qldamage := 3.057313905\ 10^5\ \pi$$

> $T3 := Tc + \dfrac{qldamage \cdot 2}{\text{alpha} \cdot \text{Pi} \cdot d3}$

$$T3 := 2875.657233$$

> $T2 := T3 + \dfrac{qldamage \cdot \ln\left(\frac{d3}{d2}\right)}{2 \cdot \text{Pi} \cdot kc}$

$$T2 := 3244.627908$$

> $T1 := T2 + CriticalGradientTemp \cdot \dfrac{(d2 - d1)}{2}$

$$T1 := 3662.166708$$

So

$$T_1 = 3662\ K$$

$$T_2 = 3244\ K$$

$$T_3 = 2876\ K$$

Solution Task (B) T(H)=3662 K