Sample: Engineering - Electric Circuits

P6.34.
We apply a 5-V-rms 20-kHz sinusoid to the input of a first order RC lowpass filter, and the output voltage in steady state is 0.5 V rms. Predict the steady stage rms output voltage after the frequency of the input signal is raised to 150 kHz and the amplitude remains constant.

Solution.

Denote: $f_1 = 20\text{ kHz}, f_2 = 150\text{ kHz}, V_0 = 5\text{ V}$ is the input rms voltage, $V_1 = 0.5\text{ V}$ is the output rms voltage at a frequency $f_1 = 20\text{ kHz}$, V_2 is the output rms voltage at a frequency $f_2 = 150\text{ kHz}$. The magnitude of the transfer function of the filter is:

$$|H(f)| = \frac{1}{\sqrt{1+(f/f_B)^2}},$$

where f_B is the half-power frequency. On the other hand, this magnitude is the ratio of magnitudes of the output and input voltages and it equals the ratio of the output rms and input rms voltages. So we have:

$$|H(f_1)| = \frac{1}{\sqrt{1+(f_1/\ _B)^2}} = \frac{V_1}{V_0};\ 1+\frac{f_1^2}{f_B^2} = \frac{V_0^2}{V_1^2}.$$

Find f_B from the expression above:

$$f_B = \frac{V_1 f_1}{\sqrt{V_0^2 - V_1^2}}.$$

Similarly, write the magnitude of the transfer function at the frequency f_2:

$$|H(f_2)| = \frac{1}{\sqrt{1+(f_2/\ _B)^2}} = \frac{V_2}{V_0}; 1+\frac{f_2^2}{f_B^2} = \frac{V_0^2}{V_2^2}.$$

Substitute in this formula the expression for f_B:

$$1+\frac{f_2^2(V_0^2 - V_1^2)}{f_1^2 V_1^2} = \frac{V_0^2}{V_2^2};\ \frac{f_2^2 V_0^2 - V_1^2(f_2^2 - f_1^2)}{f_1^2 V_1^2} = \frac{V_0^2}{V_2^2};$$

$$V_2 = \frac{V_0 V_1 f_1}{\sqrt{f_2^2 V_0^2 - V_1^2(f_2^2 - f_1^2}} = 0.067\text{ V rms}.$$

Answer: 0.067 V rms.

P6.81.
Consider the parallel resonant circuit shown in Figure 6.29 on page 317. Determine the L and C values, given $R = 1\text{ k}\Omega$, $f_0 = 10\text{ MHz}$, and $B = 500\text{ kHz}$. If $\boldsymbol{I} = 10^{-3}\angle 0^\circ\text{ A}$, draw a phasor diagram showing the currents through each of the elements in the circuit at resonance.

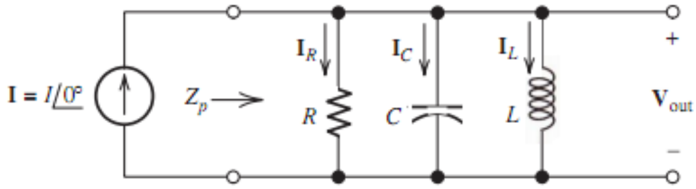


Figure 6.29 The parallel resonant circuit.

Solution.

The bandwidth of the circuit is given by:

$$B = \frac{f_0}{Q_p},$$

where Q_p is the quality factor. Find Q_p :

$$Q_p = \frac{f_0}{B}.$$

On the other hand, the quality factor is given by:

$$Q_p = \frac{R}{2\pi f_0 L}.$$

Considering the two formulas above, we have:

$$\frac{f_0}{B} = \frac{R}{2\pi f_0 L}; \quad L = \frac{BR}{2\pi f_0^2} = 0.796\ \mu\text{H}.$$

The resonant frequency f_0 and the L and C values are related by the following equation:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

Find the value of the capacitance of the circuit from this equation:

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 f_0^2}\frac{2\pi f_0^2}{BR} = \frac{1}{2\pi BR} = 0.318\ \text{nF}.$$

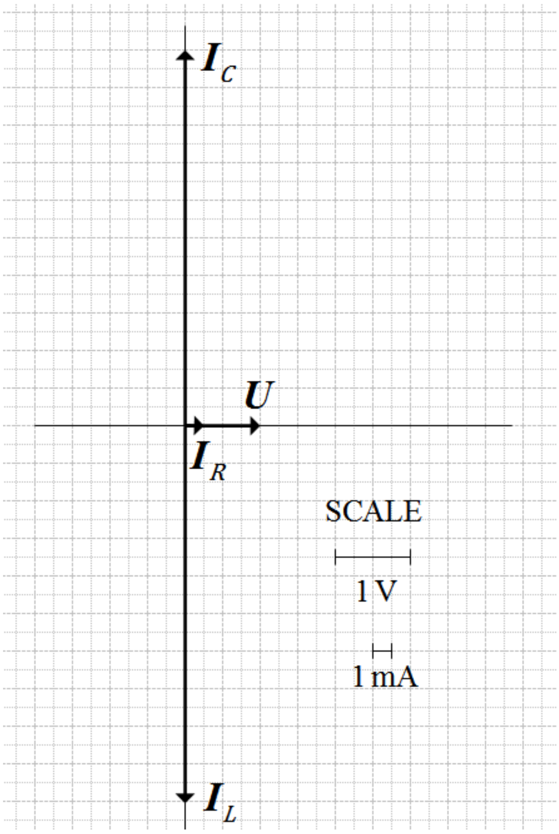
According to Ohm's law find the voltages across each of the elements in the circuit:

$$\boldsymbol{U}_R = \boldsymbol{U}_C = \boldsymbol{U}_L = \boldsymbol{I}R = 1\angle 0^\circ\ \text{V}.$$

According to Ohm's law find the currents through each of the elements in the circuit at resonance:

$$\boldsymbol{I}_R = \frac{\boldsymbol{U}}{R} = 10^{-3}\angle 0^\circ\ \text{A} = 1\angle 0^\circ\ \text{mA}, \quad \boldsymbol{I}_C = j2\pi f_0 C\boldsymbol{U} = 20\angle 90^\circ\ \text{mA}, \quad \boldsymbol{I}_L = \frac{-j\boldsymbol{U}}{2\pi f_0 L} = 20\angle -90^\circ\ \text{mA}.$$

Draw the phasor diagram from the known currents.



Answer: $L = 0.796\ \mu\text{H},\ C = 0.318\ \text{nF}.$

P6.95. Other combination of R, L, and C have behaviors similar to that of the series resonant circuit. For example, consider the circuit shown in Figure P6.95.

a. Derive an expression for the resonant frequency of this ciruit. (We have defined the resonant frequency to be the frequency for which the impedance is purely resistive).

b. Compute the resonant frequency, given $L = 1\ \text{mH},\ R = 1000\ \Omega,$ and $C = 0.25\ \mu\text{F}.$

c. Use MATLAB to obtain a plot of the impedance magnitude of this circuit for f ranging from 95 to 105 percent of the resonant frequency. Compare the result with that of a series RLC circuit.

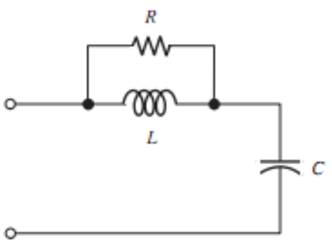


Figure P6.95

Solution.

a. This circuit consists of two parts connected in series. The first part has two elements R and L, connected in parallel, so the impedance of this part is $\frac{1}{\frac{1}{R}+\frac{1}{j\omega L}}=\frac{j\omega RL}{R+j\omega L}$. The second part has a single element C, so the impedance of this part is $\frac{-j}{\omega C}$. The impedance of the circuit is:

$$Z(w)=\frac{j\omega RL}{R+j\omega L}-\frac{j}{\omega C}.$$

Find the imaginary part of the impedance:

$$\operatorname{Im}[Z(\omega)]=\operatorname{Im}\left[\frac{j\omega RL(R-j\omega L)}{(R+j\omega L)(R-j\omega L)}-\frac{j}{\omega C}\right]=\frac{\omega R^2 L}{R^2+(\omega L)^2}-\frac{1}{\omega C}.$$

The imaginary part of the impedance equals zero at the resonant frequency:

$$\frac{\omega_0 R^2 L}{R^2+(\omega_0 L)^2}-\frac{1}{\omega_0 C}=0;\ \omega_0=\frac{R}{\sqrt{L(R^2C-L)}};f_0=\frac{\omega_0}{2\pi}=\frac{R}{2\pi\sqrt{L(R^2C\quad L)}}.$$

b. Compute the resonant frequency, given $L=1\ \text{mH}$, $R=1000\ \Omega$, and $C=0.25\ \mu\text{F}$.

$$f_0=\frac{R}{2\pi\sqrt{L(R^2C-L)}}=10.09\ \text{kHz}.$$

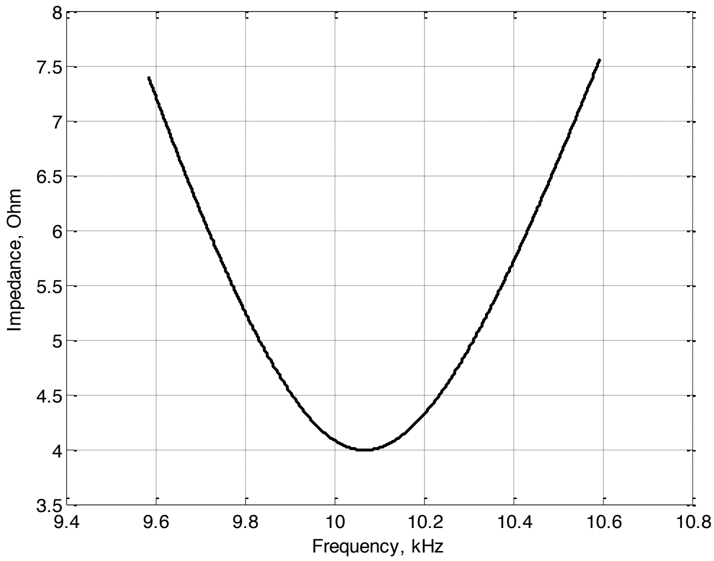
c. Write the impedance of the circuit in the following form:

$$Z(f)=\frac{j2\pi fRL}{R+j2\pi fL}-\frac{j}{2\pi fC}.$$

A MATLAB m-file that produces the plot of the impedance magnitude of the circuit is:

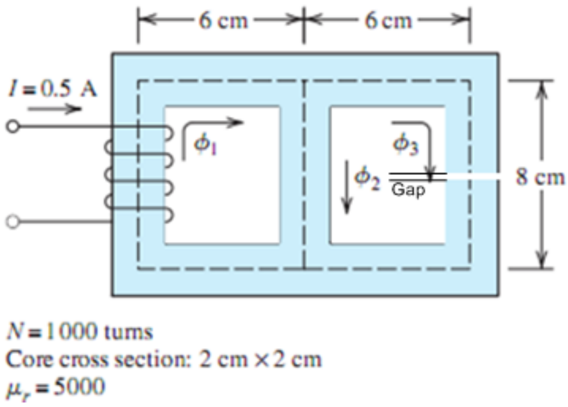
```
R=1000; L=1e-3; C=0.25e-6;
%The resonant frequency
f0=R/(2*pi*sqrt(L*(R^2*C-L)));
%The frequency interval
f=[0.95*f0:1e-4*f0:1.05*f0];
%The impedance of the circuit
Z=2*pi*i*f*R*L./(R+2*pi*i*f*L)-i./(2*pi*f*C);
%Display frequency in kHz
f=f./1000;
%Create the plot
plot(f,abs(Z),'k-','linewidth',2);
grid on;
xlabel('Frequency, kHz');
ylabel('Impedance, Ohm')
```

The plot is shown in the figure below.



The graph of the circuit impedance near the resonant frequency f_0 behaves as a graph of the input impedance of a series RLC circuit near its resonant frequency.
Answer: $f_0 = 10.09$ kHz.

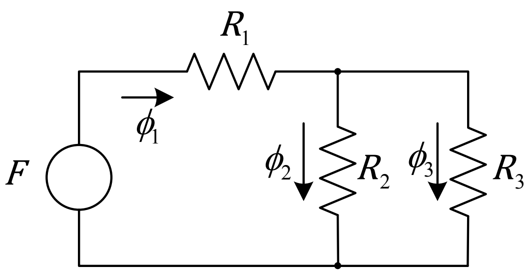
P15.30. Repeat problem P15.29 if a gap having a length of 0.5 cm is cut in the right-hand leg of the core. Account for fringing by adding the gap length to each of the cross-sectional dimensions. If you have worked Problem P15.29, prepare a table comparing the fluxes with and without gap. Comment on the results.



Solution.

The magnetic circuit is shown in the figure below. First, we find the reluctances of the parts. For the left path, we have:

$$R_1 = \frac{l_1}{\mu_0 \mu_r A_{core}} = \frac{(6+8+6)\times 10^{-2}}{4\pi\times 10^{-7}\times 5000\times 2\times 10^{-2}\times 2\times 10^{-2}} = 7.958\times 10^4 \text{ A}\cdot\text{turns / Wb.}$$



Similarly, the reluctance of the central path is:

$$R_2 = \frac{l_2}{\mu_0 \mu_r A_{core}} = \frac{8\times 10^{-2}}{4\pi\times 10^{-7}\times 5000\times 2\times 10^{-2}\times 2\times 10^{-2}} = 3.183\times 10^4 \text{ A}\cdot\text{turns / Wb.}$$

The reluctance of the left path is the sum of the reluctance of the iron core plus the reluctance of a gap. We take fringing into account by adding the gap length to its width and depth in computing area of the gap, so the area of the gap is:

$$A_{gap} = (2+0.5)\times 10^{-2}\times(2+0.5)\times 10^{-2}\text{ m} = 6.25\times 10^{-4}\text{ m}^2.$$

Then the reluctance of the left part is given by:

$$R_3 = R_{gap} + R_{core} = \frac{l_{gap}}{\mu_0 A_{gap}} + \frac{l_{core}}{\mu_0 \mu_r A_{core}} = \frac{0.5\times 10^{-2}}{4\pi\times 10^{\ 7}\times 6.25\times 10^{-4}} +$$

$$+\frac{(6+8+6-0.5)\times 10^{-2}}{4\pi\times 10^{-7}\times 5000\times 2\times 10^{-2}\times 2\times 10^{-2}} = 6.366\times 10^6 + 7.759\times 10^4 = 6.444\times 10^6 \text{ A}\cdot\text{turns / Wb.}$$

The magnetomotive force is given by:

$$F = NI = 1000\times 0.5 = 500 \text{ A}\cdot\text{turns.}$$

The reluctances R_2 and R_3 are connected in parallel. The parallel combination of R_2 and R_3 is connected in series with the reluctance R_1. Reluctances are analogous to resistances. Then the total reluctance is given by:

$$R_{total} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 7.958\times 10^4 + \frac{3.183\times 10^4\times 6.444\times 10^6}{3.183\times 10^4 + 6.444\times 10^6} = 1.113\times 10^5 \text{ A}\cdot\text{turns / Wb.}$$

The flux ϕ_1 is:

$$\phi_1 = \frac{F}{R_{total}} = \frac{500}{1.113\times 10^5} = 4.494 \text{ mWb.}$$

Fluxes are analogous to currents. Thus, we use the current-division principle to determine the fluxes ϕ_2 and ϕ_3 :

$$\phi_2 = \phi_1 \frac{R_3}{R_2 + R_3} = \frac{4.494\times 10^{-3}\times 6.444\times 10^6}{3.183\times 10^4 + 6.444\times 10^6} = 4.472 \text{ mWb;}$$

$$\phi_3 = \phi_1 \frac{R_2}{R_2 + R_3} = \frac{4.494\times 10^{-3}\times 3.183\times 10^4}{3.183\times 10^4 + 6.444\times 10^6} = 22.09\ \mu\text{Wb.}$$

Answer: $\phi_1 = 4.494\text{ mWb},\ \ \phi_2 = 4.472\text{ mWb},\ \ \phi_3 = 22.09\ \mu\text{Wb}.$

P15.64. A voltage source V_s is to be connected to a resistive load $R_L = 10\,\Omega$ by a transmission line having a resistance R_{line} $10\,\Omega$, as shown in Figure P15.64. In part (a) of the figure, no transformers are used. In part (b) of the figure, one transformer is used to step up the source voltage at the sending end of the line, and another transformer is used to step the voltage back down at the load. For each case, determine the power delivered by the source; the power dissipated in the line resistance; the power delivered to the load; and the efficiency, defined as the power delivered to the load as a percentage of the source power.

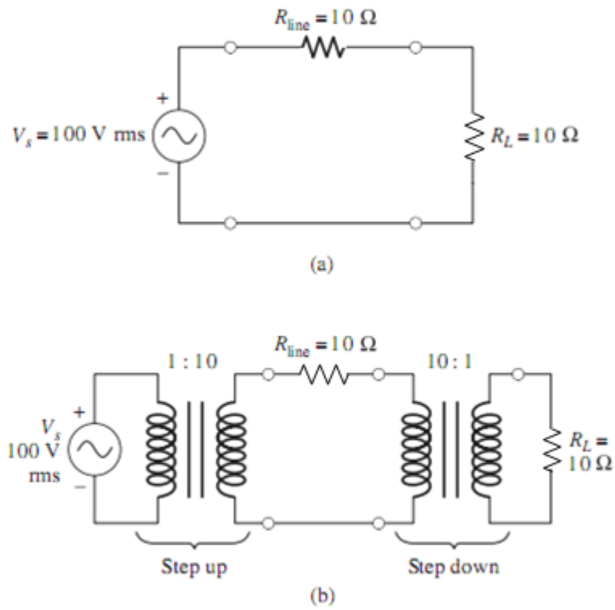
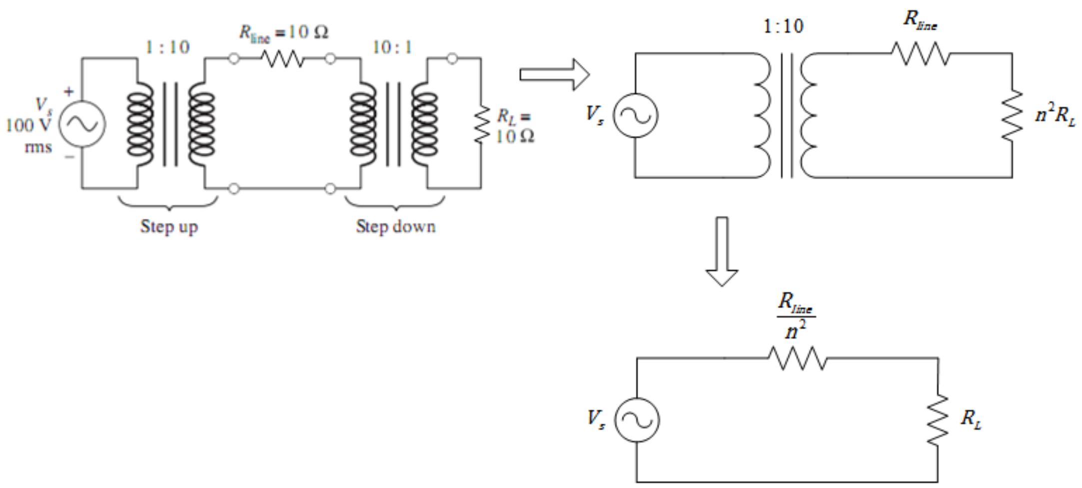


Figure P15.64

Solution.

We denote $n = 10$. Transform the circuit shown in Figure P15.64 (b), as shown in the figure below. We used the formula of impedance transformation two times to get an equivalent circuit. Thus, the current through the voltage source is



-for case (a): $I = \dfrac{V_s}{R_{line} + R_L}$;

-for case (b): $I = \dfrac{V_s}{\dfrac{R_{line}}{n^2} + R_L}$.

Both circuits are purely resistive, so the power factor $\cos(\theta)$ equals 1. Determine the power delivered by the source

- for case (a): $P_s = V_s I = \dfrac{V_s^2}{R_{line} + R_L} = 500\text{ W}$;

- for case (b): $P_s = V_s I = \dfrac{V_s^2}{\dfrac{R_{line}}{n^2} + R_L} = 990.1\text{ W}$.

Find the power dissipated in the line resistance

- for case (a): $P_{line} = I^2 R_{line} = \dfrac{V_s^2 R_{line}}{(R_{line} + R_L)^2} = 250\text{ W}$;

- for case (b): $P_{line} = I^2 R_{line} = \dfrac{V_s^2 \dfrac{R_{line}}{n^2}}{\left(\dfrac{R_{line}}{n^2} + R_L\right)^2} = 9.8\text{ W}$.

Calculate the power delivered to the load

- for case (a): $P_L = I^2 R_L = \dfrac{V_s^2 R_L}{(R_{line} + R_L)^2} = 250\text{ W}$;

- for case (b): $P_L = I^2 R_L = \dfrac{V_s^2 R_L}{\left(\dfrac{R_{line}}{n^2} + R_L\right)^2} = 980.3\text{ W}$.

Determine the efficiency

- for case (a): $\eta = \dfrac{P_L}{P_s} = \dfrac{V_s^2 R_L}{(R_{line}+R_L)^2} \dfrac{R_{line}+R_L}{V_s^2} \times 100\% = \dfrac{R_L}{R_{line}+R_L} \times 100\% = 50\%;$

- for case (b): $\eta = \text{——} = \dfrac{V_s^2 R_L}{\left(\dfrac{R_{line}}{n^2}+R_L\right)^2} \dfrac{\left(\dfrac{R_{line}}{n^2}+R_L\right)}{V_s^2} \times 100\% = \dfrac{R_L}{\dfrac{R_{line}}{n^2}+R_L} \times 100\% = 99\%.$

Thus, we can see that the circuit (b) provides more efficiency and more power to the load than the circuit (a), so the circuit (b) is energetically preferable.

Answer: $P_s = 500\ \text{W}\ (a);\ P_s = 990.1\ \text{W}\ (b);\ P_{line} = 250\ \text{W}\ (a);\ P_{line} = 9.8\ \text{W}\ (b);$

$P_L = 250\ \text{W}\ (a);\ P_L = 980.3\ \text{W}\ (b);\ \ = 50\%\ (a);\ \eta = 99\%\ (b).$

P16.50. A separately excited dc motor (see the equivalent circuit shown in Figure 16.21 on page 788) has $R_A = 1.3\,\Omega$ and $V_T = 220\ \text{V}$. For an output power of 3 hp, $n_m = 950\ \text{rpm}$ and $I_A = 12.2\ \text{A}$. The field current remains constant for all parts of this problem.

a. Find the developed power, developed torque, power lost in R_A, and the rotational losses.

b. Assuming that the rotational power loss is proportional to speed, find the no-load speed of the motor.

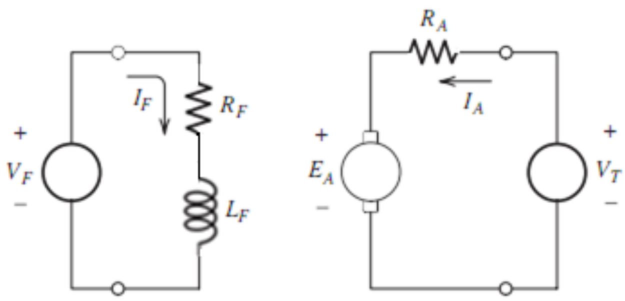


Figure 16.21 Equivalent circuit for a separately excited dc motor. Speed can be controlled by varying either source voltage (V_F or V_T).

Solution.

a. According to Ohm's law we write the following expression and find E_A:

$$V_T = R_A I_A + E_A;\ E_A = V_T - R_A I_A = 204.14\ \text{V}.$$

The developed power is given by:

$$P_{dev} = E_A I_A = 2.49\ \text{kW}.$$

Determine the angular frequency:

$$\omega_m = \frac{2\pi n_m}{60} \quad \frac{\pi n_m}{30}.$$

Calculate the developed torque:

$$T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{30 E_A I_A}{\pi n_m} = 25.03\ \text{Nm}.$$

Find the power lost in R_A:

$$P_{arm-loss} = I_A^2 R_A = 193.49 \text{ W}.$$

The output power in watts is:

$$P_{out} = 3 \text{ hp} \times 746 \frac{\text{W}}{\text{hp}} = 2.24 \text{ kW}.$$

Find the rotational losses from the law of conservation of energy:

$$_{rot} = P_{dev} - \ _{out} = 252.5 \text{ W}.$$

b. In the no-load mode $P_{dev\ no-load} = P_{rot\ no\ \ load}$. Denote the angular frequency in the no-load mode as ω_1. $P_{rot\ no-load} = \frac{P_{rot}\omega_1}{\omega_m}$, because the rotational power loss is proportional to speed, hence it proportional to angular frequency. So we have: $P_{dev\ no-load} = \frac{P_{rot}\omega_1}{\omega_m}$. On the other hand, $P_{dev\ no-load}$ is given by:

$$P_{dev\ no-load} = \frac{\omega_1 K\phi(V_T - K\phi\omega_1)}{R_A}.$$

$K\phi$ is a constant, because K is a constant for the motor and the flux ϕ is a constant due to the unchanged field current. Find $K\phi$:

$$K\phi = \frac{E_A}{\omega_m} = 2.052 \frac{\text{V}\cdot\text{s}}{\text{rad}}.$$

Write the expression and find ω_1 from it:

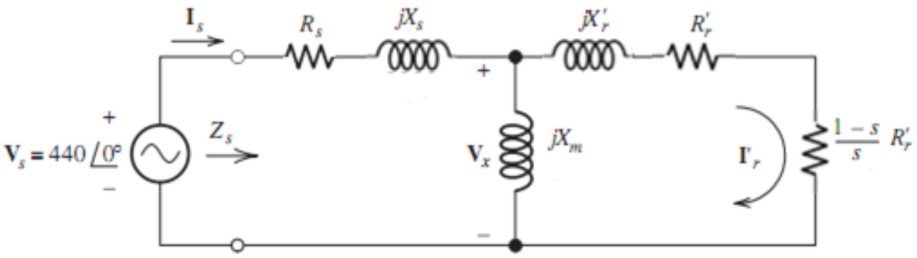
$$\frac{\omega_1 K\phi\ V_T - K\phi\omega_1)}{_A} = \frac{P_{rot}\omega_1}{\omega_m};\ \omega_1 = \frac{K\phi\omega_m V_T - P_{rot}R_A}{(K\phi)^2\omega_m} = 106.429 \text{ rad/s}.$$

Calculate the no-load speed:

$$n_{no-load} = \frac{60}{2\pi}\omega_1 = 1016 \text{ rpm}.$$

Answer: $P_{dev} = 2.49 \text{ kW}$, $T_{dev} = 25.03 \text{ Nm}$, $P_{arm-loss} = 193.49 \text{ W}$, $P_{rot} = 252.5 \text{ W}$, $n_{no-load} = 1016 \text{ rpm}$.

P17.25. A certain six-pole 440-V-rms 60-Hz three-phase delta-connected induction motor has $R_s = 0.08\,\Omega$, $R_r' = 0.06\,\Omega$, $X_s = 0.20\,\Omega$, $X_r' = 0.15\,\Omega$, $X_m = 7.5\,\Omega$. Under load, the machine operates with a slip of 4 percent and has rotational losses of 2 kW. Determine the power factor, output power, copper losses, output torque, and efficiency.



Solution.

The equivalent circuit for one phase of the motor is shown in figure above. Denote $R_1 = \frac{1 \quad s}{s} R_r' = \frac{1-0.04}{0.04} \times 0.06 = 1.44\,\Omega$. Find the impedance seen by the source:

$$Z_s = R_s + jX_s + \frac{jX_m(jX_r' + R_r' + R_1)}{jX_m + jX_r' + R_r' + R_1} = 1.468 + j0.619 = 1.59\angle 22.87^\circ\ \Omega.$$

The power factor is the cosine of the impedance angle:

$$\cos(\theta) = \cos(22.87^\circ) = 0.921.$$

For a delta-connected machine, the phase voltage is equal to the line voltage, which is specified to be 440 V rms. The phase current is given by:

$$\boldsymbol{I}_s = \frac{\boldsymbol{V}_s}{\boldsymbol{Z}_s} = 276.10\angle -22.87^\circ \text{ A rms.}$$

Find the current $\boldsymbol{I}_r'$ using the current-division principle:

$$\boldsymbol{I}_r' = \frac{\boldsymbol{I}_s jX_m}{jX_m + jX_r' + R_r' + R_1} = 265.63\angle -11.77^\circ \text{ A rms.}$$

The developed power is given by:

$$P_{dev} = 3R_1(I_r')^2 = 304.81\,\text{kW}.$$

The output power is:

$$P_{out} = P_{dev} - P_{rot} = 304.81 - 2 = 302.81\,\text{kW}.$$

The copper losses are:

$$P_{cop.loss} = 3R_s I_s^2 + 3R_r'(I_r')^2 = 31.00\,\text{kW}.$$

Find the angular frequency ω_m:

$$s = \frac{\omega_s - \omega_m}{\omega_s};\ \omega_m = \omega_s(1-s) = 2\pi \times 60 \times 0.96 = 361.91\,\text{rad/s}.$$

Calculate the output torque:

$$T_{out} = \frac{P_{out}}{\omega_m} = 836.70\ \text{Nm}.$$

Determine the input power:

$$P_{in} \quad 3V_s I_s \cos(\theta) = 335.81\,\text{kW}.$$

Calculate the efficiency:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 90.17\%.$$

Answer: $\cos(\theta) = 0.921$, $P_{out} = 302.81\,\text{kW}$, $P_{cop.loss} = 31.00\,\text{kW}$, $T_{out} = 836.70\ \text{Nm}$, $=90.17\%$.