Sample: Electric Circuits - Circuits with Capacitors

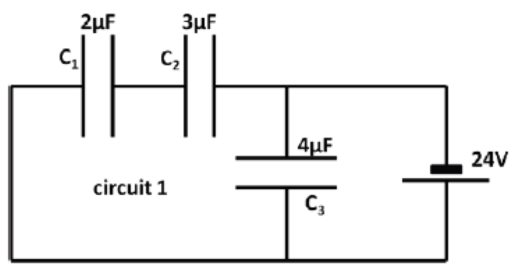
World of the electron: coursework 1

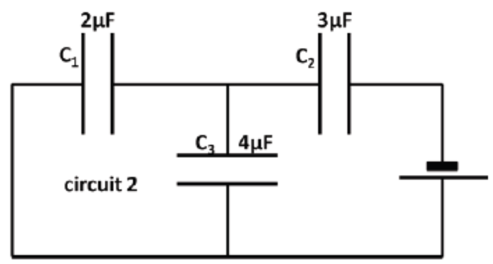
Show all workings, including diagrams where appropriate.

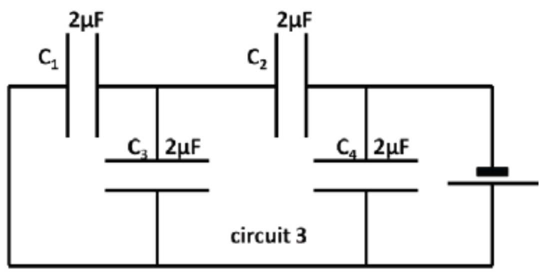
1) For each of the circuits shown below, calculate the total equivalent capacitance of the circuit.
[10 marks]

2) For circuit 1, calculate the total amount of energy stored by this equivalent capacitance.
[2 marks]

3) For circuit 1, calculate the charge on each capacitor and the potential difference across each capacitor.
[8 marks]







1.

$$C_1 = 2\ \mu F$$
$$C_2 = 3\ \mu F$$
$$C_3 = 4\ \mu F$$

I circuit:

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 C_2}$$
$$C_{12} = \frac{C_1 C_2}{C_1 + C_2}$$

Now for the whole circuit:

$$C = C_3 + C_{12} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = \frac{C_3(C_1 + C_2) + C_1 C_2}{C_1 + C_2} = \frac{C_1 C_3 + C_2 C_3 + C_1 C_2}{C_1 + C_2}$$
$$= \frac{2\ \mu F \times 4\ \mu F + 3\ \mu F \times 4\ \mu F + 2\ \mu F \times 3\ \mu F}{2\ \mu F + 3\ \mu F} = \frac{8 + 12 + 6}{5}\mu F = \frac{26}{5}\mu F$$
$$= 5.2\ \mu F$$

II circuit:

$$C_{13} = C_1 + C_3$$
$$\frac{1}{C} = \frac{1}{C_{13}} + \frac{1}{C_2} = \frac{1}{C_1 + C_3} + \frac{1}{C_2} = \frac{C_2 + C_1 + C_3}{C_2(C_1 + C_3)}$$
$$C = \frac{C_2(C_1 + C_3)}{C_1 + C_2 + C_3} = \frac{3(2+4)}{2+3+4}\mu F = \frac{18}{9}\mu F = 2\mu F$$

III circuit:

$$C_1 = 2\ \mu F$$
$$C_2 = 3\ \mu F$$
$$C_3 = 4\ \mu F$$
$$C_4 = 2\ \mu F$$
$$C_{13} = C_1 + C_3$$
$$\frac{1}{C_{123}} = \frac{1}{C_{13}} + \frac{1}{C_2} = \frac{1}{C_1 + C_3} + \frac{1}{C_2} = \frac{C_2 + C_1 + C_3}{C_2(C_1 + C_3)}$$
$$C_{123} = \frac{C_2(C_1 + C_3)}{C_1 + C_2 + C_3}$$
$$C_{1234} = C_{123} + C_4 = \frac{C_2(C_1 + C_3)}{C_1 + C_2 + C_3} + C_4 = \frac{C_1 C_2 + C_2 C_3 + C_1 C_4 + C_2 C_4 + C_3 C_4}{C_1 + C_2 + C_3}$$
$$= \frac{2 \times 3 + 3 \times 4 + 2 \times 2 + 2 \times 2 + 4 \times 2}{2 + 3 + 4} = \frac{6 + 12 + 4 + 4 + 8}{9}\mu F$$
$$= \frac{34}{9}\mu F = 3.77\ \mu F$$

2.

$$C_{1circuit} = 5.2\ \mu F$$

Total amount of energy stored:

$$W = \frac{CU^2}{2} = \frac{5.2\ \mu F \times (24\ V)^2}{2} = \frac{5.2\ \mu F \times 576\ V^2}{2} = 1497.6\ J$$

3.

Let’s use charge conservation law:

$$q_1 + q_2 + q_3 = 0$$

$$E = U_3 = U_1 + U_2 = 24\ V$$

By definition:

$$q_3 = C_3 U_3 = 4\ \mu F \times 24\ V = 96 \times 10^{-6} C$$

$$q_2 = C_2 U_2$$

$$q_1 = C_1 U_1$$

Let’s also use formula

$$U_2 = E - U_1$$

From first we get:

$$q_1 = -q_2 - q_3 = -C_2 U_2 - q_3 = -C_2(E - U_1) - q_3 = C_1 U_1$$

From where we get U_1 :

$$C_1 U_1 = -C_2 E + C_2 U_1 - q_3$$

$$U_1 (C_2 - C_1) = C_2 E + q_3$$

$$U_1 = \frac{C_2 E + q_3}{C_2 - C_1} = \frac{3\mu F \times 24V + 96\ C}{(3-2)\mu F} = 168\ V$$

$$U_2 = E - U_1 = 24V - 168V = -144V$$

Minus sign shows us that charge would be negative.

$$q_1 = C_1 U_1 = 2\mu F \times 168V = 336\ \times 10^{-6} C$$

$$q_2 = C_2 U_2 = 3\ \mu F \times -144\ V = -432\ \times 10^{-6} C$$