Sample: Calculus - Rules for Derivatives

1. Find the derivative for the following:
a. $y = x^2 e^x$
b. $y = (e^x + 2)^{\frac{3}{2}}$
c. $y = e^{-3x}$
d. $y = \frac{e^x - e^{-x}}{2}$

Solution

a. Use the product rule:
$y' = 2xe^x + x^2 e^x$
b. Using the chain rule:
$$y' = \frac{3}{2}(e^x + 2)^{\left(\frac{3}{2}-1\right)} \cdot e^x = \frac{3}{2} e^x \sqrt{e^x + 2}$$
c. Let use the chain rule:
$y' = -3e^{-3x}$
d. Let factor out the constant $\frac{1}{2}$:
$$y = \frac{1}{2}(e^x - e^{-x})$$
Differentiate the sum term by term, using chain rule for the second term:
$$y' = \frac{1}{2}\left(e^x - (-e^{-x})\right) = \frac{1}{2}(e^x + e^{-x})$$

Answer

a. $2xe^x + x^2 e^x$
$b. \frac{3}{2} e^x \sqrt{e^x + 2}$
c. $-3e^{-3x}$
d. $\frac{1}{2}(e^x + e^{-x})$

2. The present value of a building in the downtown area is given by the function

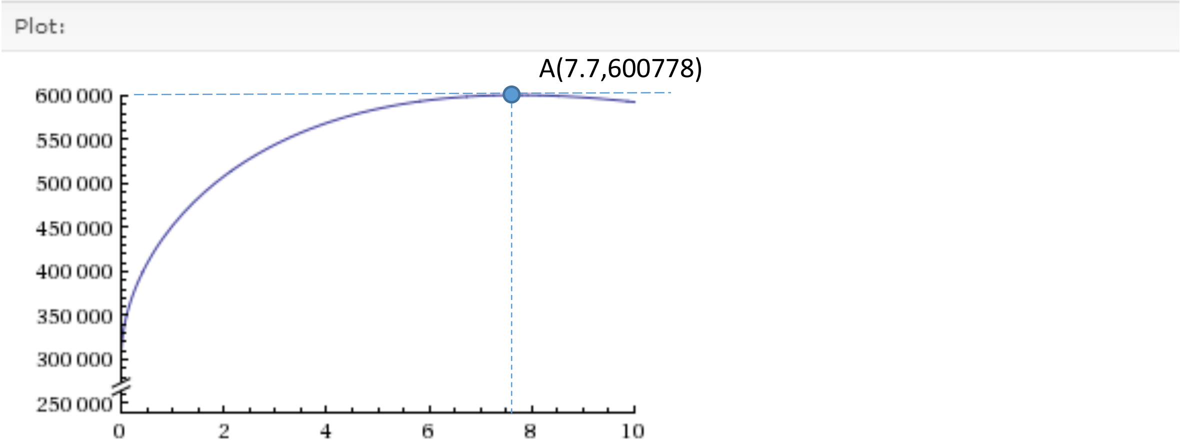
$$P(t) = 300{,}000 e^{-0.09t + \frac{\sqrt{t}}{2}} \; for \; 0 \le t \le 10$$

Find the optimal present value of the building. (Hint: Use a graphing utility to graph the function, $P(t)$, and find the value of t_0 that gives a point on the graph, $(t_0, P(t_0))$, where the slope of the tangent line is 0.)

Solution

Graph the function and find the point, where the slope of the tangent line is 0 (line is parallel to the x-axis):

plot	$P(t) = 300\,000\, e^{\frac{\sqrt{t}}{2} - 0.09t}$	$t = 0$ to 10



Substitute $t = 7.7$ to calculate the optimal present value of the building:

$$P(t) = 300{,}000e^{-0.09\cdot 7.7+\frac{\sqrt{7.7}}{2}} \approx 600{,}778$$

Answer

600,778

3. Find the equation of the line tangent to

$$f(x) = xe^{-x},$$

at the point where $x = 0$. What does this tell you about the behavior of the graph when $x = 0$?

Solution

$y - f(x_0) = m(x - x_0), where\ m = f'(x_0)$

$f(0) = 0 \cdot 1 = 0$

$f'(x) = e^{-x} - xe^{-x}$

$f'(0) = 1 - 0 \cdot 1 = 1$

Substitute all values in the equation:

$y - 0 = 1(x - 0)$

$y = x$

It means that the function is increasing and curve is rising up to the right.

Answer

$y = x$