Sample: Discrete Mathematics - Applications of Discrete Math

1. The following is a message in ASCII. What does it say?

0100100100100000011101110110000101101110011101000010000001110100011011110010000001100111011001010111010000100000011000010110111000100000010000010010000001101001011011100010000001110100011010000110100101110011001000000110001101101100011000010111001101110011

Solution:

Each letter in binary form takes 8 bits (1 byte). First, we need to break the original binary code octets of 8 bits:

01001001 00100000 01110111 01100001 01101110 01110100 00100000 01110100
01101111 00100000 01100111 01100101 01110100 00100000 01100001 01101110
00100000 01000001 00100000 01101001 01101110 00100000 01110100 01101000
01101001 01110011 00100000 01100011 01101100 01100001 01110011 01110011

Next, we need to translate the number of each octet of the binary to decimal form (byte per character):

73 32 119 97 110 116 32 116 111 32 103 101 116 32 97 110 32 65 32 105 110 32 116 104 105 115 32 99 108 97 115 115

Then we need to translate each number into a character according to ASCII Table. (one number - one character). For example, the number 73 is the letter "I", the number 32 is the character "_", and so on.

Message: "I want to get an A in this class".

2. The following is a message coded in ASCII using one byte per character and then represented in hexadecimal notation. What is the message?

4469736372657465204D617468656D61746963732069732043F4F4C21

Solution:

Each letter in binary form takes 8 bits (1 byte). First, we need to break the original hexadecimal code on parts of 2 characters (2 characters of hexadecimal code = 1 byte of binary code):

44 69 73 63 72 65 74 65 20 4D 61 74 68 65 6D 61 74 69 63 73 20 69 73 20 43 4F 4F 4C 21

Next, we need to translate the number of each part of the hexadecimal code to decimal form (2 characters per one number):

68 105 115 99 114 101 116 101 32 77 97 116 104 101 109 97 116 105 99 115 32 105 115 32 67 79 79 76 33

Then we need to translate each number into a character according to ASCII Table. (one number - one character). For example, the number 68 is the letter "D", the number 105 is the character "i", and so on.

Message: "Discrete Mathematics is COOL!"

3. Convert each of the following binary representations to its equivalent base-ten representation. Show your work step by step.

- 10001
- 0000111
- 01011100
- 110101

Solution:

Every bit responsible for the coefficient near a power of two, with a power of two increases from right to left (... $\leftarrow 2^5 \leftarrow 2^4 \leftarrow 2^3 \leftarrow 2^2 \leftarrow 2^1 \leftarrow 2^0$):

10001:

2^4	2^3	2^2	2^1	2^0
1	0	0	0	1

$$10001_2 = 1\cdot 2^4 + 0\cdot 2^3 + 0\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 = 16 + 1 = 17$$

0000111:

2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	0	0	1	1	1

$$0000111_2 = 0\cdot 2^6 + 0\cdot 2^5 + 0\cdot 2^4 + 0\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 = 4 + 2 + 1 = 7$$

01011100:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	1	1	0	0

$$01011100_2 = 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$$
$$= 64 + 16 + 8 + 4 = 92$$

110101:

2^5	2^4	2^3	2^2	2^1	2^0
1	1	0	1	0	1

$$110101_2 = 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 32 + 16 + 4 + 1$$
$$= 53$$

4. Convert each of the following base-ten representations to its equivalent two's complement in 7 bits. Show all of your work.

- 12
- -2
- -8
- 22
- 0

Define the highest and lowest integer that can be represented in this 7-bit two's complement representation.

Solution:

- First, we must divide the number by 2, while the remainder of the division is not equal to 1.
- Then we must write the remainders of the division in a separate column, and to get the number in binary form we must write the remainder of the division in the reverse order (bottom to top) and fill the empty places to 7 bits with zeros.

Additional steps for negative numbers:

- Next is inverting the bits, switching the 1's and 0's (inverse code).
- The last step is to add 1 to the number.

Number 12:

Integer part	Remainder
12 div 2 = 6	12 mod 2 = **0**

6 div 2 = 3	6 mod 2 = **0**
3 div 2 = 1	3 mod 2 = **1**
1 div 2 = 1	1 mod 2 = **1**

$12_{10} = 1100_2 = 0001100$

Number -2:

Integer part	Remainder
2 div 2 = 1	2 mod 2 = **0**
1 div 2 = 0	1 mod 2 = **1**

$2_{10} = 10_2 = 0000010$

inverse code: 1111101

adding 1 to our number: $1111101 + 0000001 = 1111110$

$-2_{10} = 1111110$

Number -8:

Integer part	Remainder
8 div 2 = 4	8 mod 2 = **0**
4 div 2 = 2	4 mod 2 = **0**
2 div 2 = 1	2 mod 2 = **0**
1 div 2 = 1	1 mod 2 = **1**

$8_{10} = 1000_2 = 0001000$

inverse code: 1110111

adding 1 to our number: $1110111 + 0000001 = 1111000$

$-8_{10} = 1111000$

Number 22

Integer part	Remainder
22 div 2 = 11	22 mod 2 = **0**
11 div 2 = 5	11 mod 2 = **1**
5 div 2 = 2	5 mod 2 = **1**
2 div 2 = 1	2 mod 2 = **0**
1 div 2 = 0	1 mod 2 = **1**

$22_{10} = 10110_2 = 0010110$

Number 0

Number 0 is always zero:

$0_{10} = 0 = 000000.$

Highest integer that can be represented in this 7-bit two's complement representation: when all 6 bits are filled units (1) except the sign bit (0): $0111111 = 63_{10}$

Lowest integer that can be represented in this 7-bit two's complement representation: when 6 bits are filled with 6 zeros (0) (sign bit is "1" that shows that the number is negative): $1000000 = -64_{10}$

Answer:

$12_{10} = 0001100$

$-2_{10} = 1111110$

$-8_{10} = 1111000$

$22_{10} = 10110_2 = 0010110$

$0 = 000000$

Highest integer: $0111111 = 63$

Lowest integer: $1000000 = -64$

5. What bit patterns are represented by the following hexadecimal notations? Show all of your work.

- 9A88
- 4AF6
- DA
- AD

Solution:

Every bit responsible for the coefficient near a power of 16, with a power of 16: 0=0000, 1=0001, 2=0010, 3=0011, 4=0100, 5=0101, 6=0110, 7=0111, 8=1000, 9=1001, A=1010, B=1011, C=1100, D=1101, E=1110, F=1111 (one character in hexadecimal notation – 4 bits of binary code):

9A88:

1001	1010	1000	1000
9	A	8	8

$$9A88_{16} = 1001101010001000$$

4AF6:

0100	1010	1111	0110
4	A	F	6

$$4AF6_{16} = 0100101011110110$$

DA:

1101	1010
D	A

$$DA_{16} = 11011010$$

AD:

1010	1101
A	D

$$AD_{16} = 10101101$$

Task 2

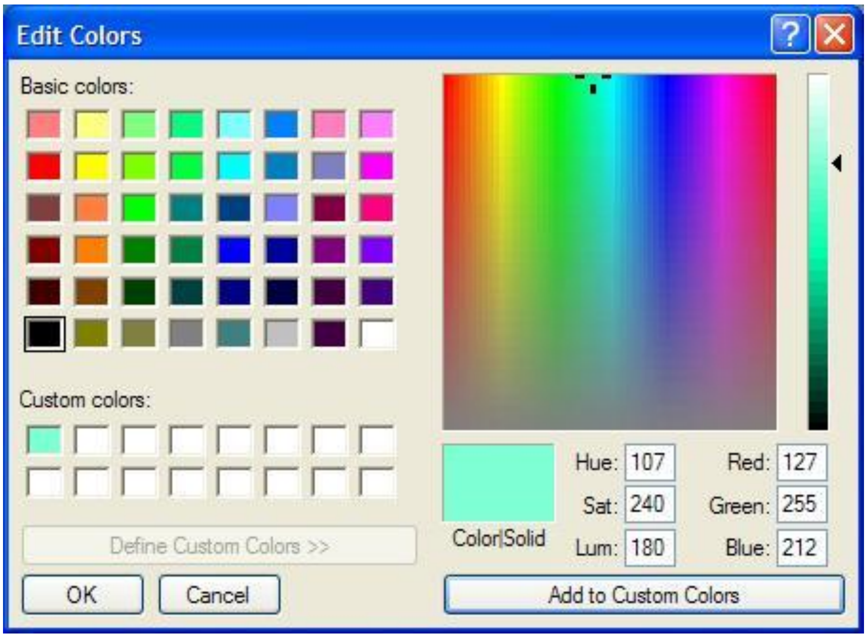
Colors for tables and Web site backgrounds must be chosen when Web pages are built. There are 17 standard color names defined in the HTML and CSS: aqua, black, blue, fuchsia, gray, grey, green, lime, maroon, navy, olive, purple, red, silver, teal, white, and yellow.

Each color has a unique hexadecimal value that consists of 6 hexadecimal digits. The hexadecimal value can be split into 3 pairs. Each pair corresponds to the intensity of the colors red, green, and blue. Then, the hexadecimal code of each of those colors is translated to an RGB color code that is triplet of decimal numbers.

For example:

Color	**Hexadecimal**	**RGB**
aquamarine	7FFFD4	127,255,212

In Paint, this is designated in the second column of numbers in the bottom right corner. The following image shows the above Paint color option for aquamarine.



Choose a color from the table below and write a few paragraphs (100–300 words) describing your solution to following exercise:

1. Translate the hexadecimal code of your choice into RGB code.
2. Open Paint, click on colors, choose edit colors, and pick define custom colors.
3. Copy each number of your triplet in the red, green, and blue boxes. Verify that the color that you have created is the one that you expected.
4. Finally, compare your color's hexadecimal code to its binary code.

http://class.aiuniv.edu/LCMSFileShareCommon/d02/9f9/9ad/2d4/497/395/0ca/30a/ae3/1ea/d4/143379_b.pdf

Color	HEX	Color	HEX
Alice Blue	F0F8FF	Dark Gray	A9A9A9
Antique White	FAEBD7	Dark Grey	A9A9A9
Aqua	00FFFF	Dark Green	006400
Aquamarine	7FFFD4	Dark Khaki	BDB76B
Azure	F0FFFF	Dark Magenta	8B008B
Beige	F5F5DC	Dark Olive Green	556B2F
Bisque	FFE4C4	Dark orange	FF8C00
Black	000000	Dark Orchid	9932CC
Blanched Almond	FFEBCD	Dark Red	8B0000
Blue	0000FF	Dark Salmon	E9967A
Blue Violet	8A2BE2	Dark Sea Green	8FBC8F
Brown	A52A2A	Dark Slate Blue	483D8B
Burly Wood	DEB887	Dark Slate Gray	2F4F4F
Cadet Blue	5F9EA0	Dark Slate Grey	2F4F4F
Chartreuse	7FFF00	Dark Turquoise	00CED1
Chocolate	D2691E	Dark Violet	9400D3
Coral	FF7F50	Deep Pink	FF1493
Corn flower Blue	6495ED	Deep Sky Blue	00BFFF
Corn silk	FFF8DC	Dim Gray	696969
Crimson	DC143C	Dim Grey	696969
Cyan	00FFFF	Dodger Blue	1E90FF
Dark Blue	00008B	Fire Brick	B22222
Dark Cyan	008B8B	Floral White	FFFAF0
Dark Golden Rod	B8860B	Forest Green	228B22

I chose the color “Antuque White”, hexadecimal code of my choice is FAEBD7. To translate the hexadecimal code of my choice into RGB code I split the number by 3 parts (each part - is the number of RGB):

$$FA \quad EB \quad D7$$

Then I converted hexadecimal number to decimal form and binary code:

$$FA_{16} = 15 \cdot 16^1 + 10 \cdot 16^0 = 250 = 11111010$$

$$EB_{16} = 14 \cdot 16^1 + 11 \cdot 16^0 = 235 = 11101011$$

$$D7_{16} = 13 \cdot 16^1 + 7 \cdot 16^0 = 215 = 1101111$$

So, the RGB number of the color “Antuque White” is (250, 235, 215). Next I found the information about this color in the Internet. Also I had compared my color's hexadecimal code to its binary code and RGB code in Paint. I found that the color that you have created is the one that I expected to see.

RGB Color: Antique white (RGB values 250, 235, 215)

Information about **RGB color 250, 235, 215**

HEX:	#FAEBD7	**CMYK:**	0.00, 0.06, 0.14, 0.02
RGB:	250, 235, 215	**CIE XYZ:**	0.82, 0.86, 0.79
HSL:	34°, 0.78, 0.91	**CIE LAB:**	94.23, 1.64, 10.64
HSV:	34°, 0.14, 0.98	**YUV:**	0.93, -0.04, 0.04